Personal Finance: Summer Packet

Due on the first day of class

Name: ______________________________

Welcome to Personal Finance!

The course you're about to take is rather different that the math classes you've taken this far in your high school careers. While this is still a math class and your skills from all your previous math classes will be useful, this course is *interdisciplinary* in nature, meaning that you will be expected to use your skills from other subjects in order to succeed. I find that running Personal Finance in this way levels the playing field, since this class is often made up of students of all mathematical skill levels, who all deserve a reasonable challenge.

Like the class as a whole, this assignment is a little bit different than a traditional math summer assignment as well. For each of the math sections, I have provided an overview of the topic and a couple model problems to try to help you. Because of all the overview, this assignment is not as long as it looks! You are also welcome to use any online resource at your disposal (I especially recommend Khan Academy). If you choose to use an online resource, please indicate on which problems you used it, so I know where you had some difficulty. It's okay to get help– in fact, I expect you to in some cases– just be honest about when and where you did.

For the reading and writing component, it should be fairly straightforward. Read and mark up the article as you go, and work thoroughly. I'm a math teacher– I'm not looking for the absolute, most proper writing– but I do expect you to organize your thoughts in a clear manner and using an appropriate academic tone. You will prepare points for discussion as well as summarize your thoughts in a brief reflection.

As of right now, Mrs. delaCruz and I are planning to split the teaching duties for this class. For me, **I plan to count this assignment as a series of homework grades. There will also be a quiz on this content within the first few days of class.** However, things often change over the summer; no matter who your teacher is in the fall, assume that this assignment will count as a grade and that the content will be assessed in some way.

You must show all work to receive full credit, and work should be completed neatly and thoroughly, preferably in pencil. In the interest of saving some paper, I didn't provide a lot of room to complete this assignment, so **please work on separate sheets of paper, and attach them to this packet before submitting.** You will find your calculator helpful at times, but please show enough setup so I can follow your work.

If you have any questions at all over the summer, please reach out to me! My email is **rcox@theproutschool.org**. I would be happy to Zoom with you on a case-by-case basis if you're having difficulty.

Have a great summer! - Mr. Cox (and Mrs. delaCruz)

1 Math content review

1.1 Manipulating functions

Evaluating functions

We can think about a function as a machine with an operating system. A function f (which is the name of our machine) is a rule (operating system) relating inputs to outputs. Every input x in the *domain* must go to exactly one output y in the *range*. It's okay to have two inputs go to the same output–we can have two computer commands for the same function, for example–but we can never have one input go to two different outputs.

We typically notate our functions algebraically by the function's name and the variable it takes as an input: $f(x)$, for example. Then we define the rule: $f(x) = x^2 + 1$. So, this function's "operating system" takes every input, squares it, and adds one. It doesn't matter what we put inside. Even this is perfectly acceptable:

$$f(\quad) = (\quad)^2 + 1$$

Which, I think, shows even more clearly that whatever I plug in to f as an input is what is applied to the function's rule:

$$f(2x) = (2x)^2 + 1 = 4x^2 + 1$$
$$f(3) = 3^2 + 1 = 10$$

We call this process *evaluating* a function.

1-4. Let $f(x) = 2x^2 + 3x - 5$. Find the following, and simplify completely.

1. $f(-1)$
2. $f(3)$
3. $f(0.5)$
4. $f(3x)$

5-8. Let $g(x) = \dfrac{1}{2x+7}$. Find the following, and simplify completely.

5. $g(-2)$
6. $g(10)$
7. $g(2t+7)$
8. $g(t^2)$

Operations with functions

For four of our operations, the rule is simple: *whatever operation is present in parentheses, do that operation between the two functions.* So, we can define addition, subtraction, multiplication, and division for functions:

$$(f+g)(x) = f(x) + g(x) \qquad (f-g)(x) = f(x) - g(x)$$
$$(fg)(x) = f(x) \cdot g(x) \qquad \left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)},\ g(x) \neq 0$$

For example, let $f(x) = x + 1$ and $g(x) = x^2 - 2x$:

$$\begin{aligned}(f+g)(x) &= f(x) + g(x) \\ &= (x+1) + (x^2 - 2x) \\ &= x^2 - x + 1\end{aligned} \qquad \begin{aligned}(f-g)(x) &= f(x) - g(x) \\ &= (x+1) - (x^2 - 2x) \\ &= 1 + 3x - x^2\end{aligned}$$

$$\begin{aligned}(fg)(x) &= f(x) \cdot g(x) \\ &= (x+1)(x^2 - 2x) \\ &= x^3 - x^2 - 2x\end{aligned} \qquad \begin{aligned}\left(\frac{f}{g}\right)(x) &= \frac{f(x)}{g(x)} \\ &= \frac{x+1}{x^2 - 2x}\end{aligned}$$

Things get slightly more complicated when we talk about *composition* of functions, defined and notated this way:

$$(f \circ g)(x) = f\big(g(x)\big)$$

Remember how we just talked about evaluating functions? This time, we're "evaluting" a function with another function. Problems 4, 7, and 8 were actually examples of function composition. I find it helpful to rewrite the problem with that substitution, so it looks like those problems, as I do below.

For example, let $f(x) = x^2 + 5$ and $g(x) = x - 1$. Find $(f \circ g)(x)$ and $(g \circ f)(x)$.

$$\begin{aligned}(f \circ g)(x) &= f(g(x)) \\ &= f(x-1) \\ &= (x-1)^2 + 5 \\ &= x^2 - 2x + 1 + 5 \\ &= x^2 - 2x + 6\end{aligned} \qquad \begin{aligned}(g \circ f)(x) &= g(f(x)) \\ &= g(x^2 + 5) \\ &= (x^2 + 5) - 1 \\ &= x^2 + 4\end{aligned}$$

9-11. For each pair of functions f and g, perform the indicated operation, and evaluate if required. Answers should be simplified fully.

9. $f(x) = x^2 + 1,\ g(x) = 2x - 3$

   a. $(f+g)(x)$ b. $(fg)(x)$ c. $(f \circ g)(x)$

10. $f(x) = x^2 + 2x - 1,\ g(x) = 2x$

   a. $(f \circ g)(x)$ b. $(g \circ f)(x)$ c. $\left(\frac{f}{g}\right)(x)$

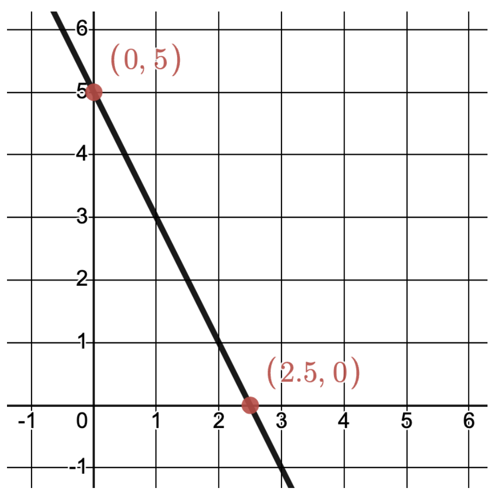
11. $f(x) = x^2 + 1,\ g(x) = 2x - 3$

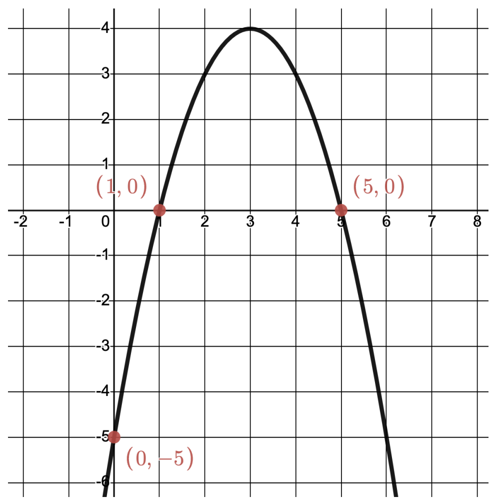
   a. $(f+g)(0)$ b. $(fg)(2)$ c. $(f \circ g)(-3)$

1.2 Analyzing functions and their graphs

Interpreting intercepts

In application problems, the *intercepts* can be useful tools for interpreting data in a real-world context. The x-*intercept* is the point $(x, 0)$ where the graph touches or crosses the x-axis. Similarly, the y-*intercept* is the point $(0, y)$ where the graph touches or crosses the y-axis. In the graphs below, the x- and y-intercepts have been labeled for you.





To find the y-intercepts, evaluate $f(0)$. To find the x-intercepts, solve the equation $f(x) = 0$. In other words, **plug in 0 for the other variable and solve.** For example, for the equation $y = 3x - 1$:

x-intercept:	Let $y = 0$	y-intercept:	Let $x = 0$
	$0 = 3x - 1$		$y = 3(0) - 1$
	$1 = 3x$		$y = -1$
	$x = \frac{1}{3}$		

So, the intercepts are $\left(\frac{1}{3}, 0\right)$ and $(0, -1)$. This can also be done on your graphing calculator, and we'll talk about that (and applications) during the year.

12-14. For each of the functions given, find all x-intercepts and y-intercepts.

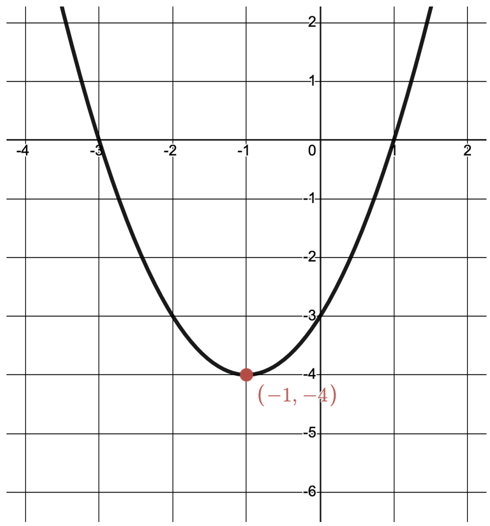
12. $y = 6x + 7$

13. $3x - 4y = 10$

14. $y = x^2 - 4$

Maxima or minima of a parabola

Extrema, the general term for maxima or minima, are also useful in application problems. This year in particular, we'll talk about cases where businesses look for ways to maximize their profit or minimize their costs. In the graph below, the minimum of the parabola is shown.



Two notes:

- There is a calculus method for finding local extrema that some of you in IB may have learned (or will learn). I won't teach that in this class but you're welcome to use it if you know.
- For now, we'll keep the numbers simple, as I haven't taught you the method on your calculator yet. We'll spend a bunch of time on this though during the first unit.

For a quadratic equation in standard form $f(x) = ax^2 + bx + c$, the x-coordinate of the max/min is given by $x = -\dfrac{b}{2a}$. Then, substitute that value into the function to find the y-coordinate. Hence, for $y = 2x^2 - 3x + 1$, the minimum point is $\left(\dfrac{3}{4}, -\dfrac{1}{8}\right)$.

15-18. Find the exact maximum or minimum value for each of the functions given.

15. $y = 3x^2 + 2x - 3$
16. $f(x) = 2 - 4x - x^2$
17. $y = x^2 + 3x - 5$
18. $g(x) = 3 + x - x^2$

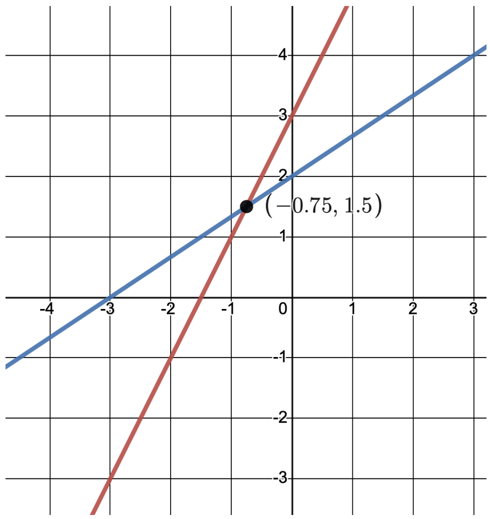
Finding the intersections of graphs

Two graphs *intersect* one another when their function values are equal. Again, there's a calculator way to do this, but we'll focus on solving them by hand for now. Simply set the functions equal to each other and solve for x.

For example, let $f(x) = 2x + 3$ and $g(x) = 3x - 5$ and find their intersection:

$$\begin{aligned} 2x + 3 &= 3x - 5 \\ -x + 3 &= -5 \\ -x &= -8 \\ x &= 8 \end{aligned}$$

Then, substitute the x-value into either function to find the y-coordinate. $f(8) = g(8) = 19$, so the point of intersection here is $(8, 19)$. Using different functions, an intersection looks like this:



19-21. For each pair of functions, determine the point(s) of intersection.

19. $\begin{cases} y = 3x + 5 \\ y = 2x - 7 \end{cases}$

20. $\begin{cases} y = \frac{1}{2}x + 1 \\ y = 3 - x \end{cases}$

21. $\begin{cases} y = 2x + 5 \\ y = 2x^2 + 1 \end{cases}$

1.3 Writing and using algebraic expressions

I get it...word problems can be challenging. Unfortunately, they are an important part of Personal Finance. Let's talk about a couple helpful tips:

- Write down any important details off to the side or in the answer space for a problem.
  - Some teachers tell students to underline or circle within the paragraph, but that doesn't fix the real problem, which is having to pick through the paragraph when you need information. **Get it out of the paragraph.**
- As you identify information, assign logical variables to the information you isolate. (C for cost, t for time, etc.)
- Sort your information into what was *given*, and what your *goal* is.

There's not much to demonstrate here, so try the following problems.

22-25. Write an expression or equation that represents the problem. Then, solve using the given conditions.

22. Emma is buying tickets for an upcoming Luke Combs concert. Ticketmaster charges her \$74.95 per ticket, plus a flat \$32.08 processing fee for the order. Write an equation for the cost C of buying t tickets, then calculate how much it will cost Emma to buy 4 tickets.

23. Brendan works for Graphic Expressions and is assessing the cost of producing senior class t-shirts. Suppose that it costs Graphic Expressions \$2.50 to buy each shirt plus \$18.14 to run the printing machine for the order. Write an equation for the expense E of producing s shirts, then calculate how much it will cost to produce 102 shirts.

24. Lauren is rewatching *Stranger Things* before Season 5 is released. For the sake of this problem, assume that Netflix releases one episode of *Stranger Things* per week (or 1 episode every 7 days). Lauren watches 3 episodes per day to catch up. At the time of writing, there are 34 episodes out.

    (a) Write an expression for the number of episodes n available d days since Lauren started watching.

    (b) Write an expression for the number of episode n Lauren has watched in d days.

    (c) After how many days will Lauren be caught up?

25. Chris and Noah are in a race. Suppose that Chris is running at 8 miles per hour. Noah, however, is running at 6 miles per hour, but gets a $\frac{3}{4}$-mile head start.

    (a) Write an expression for the distance d that Chris covered after t hours.

    (b) Write an expression for the distance d that Noah covered after t hours.

    (c) After how many *minutes* does Chris catch up to Noah?

The second part of this assignment begins on the next page.

2 Talking about personal finance

For this part of the summer assignment, read the two articles attached (hyperlinks also included below). These are your copies to use, so I highly recommend marking them up by underlining or highlighting important points and making notes in the margins with any questions or thoughts you have as you're reading. (This is also good practice for skills that will be useful when you get to college.) Then, answer the following questions.

- For the questions about each individual article, just jot down some bullet points about your thoughts, reactions, and questions while reading. We'll use these as discussion points in class.
- For the synthesis question at the end, answer this in more formal academic writing. Use full sentences, paragraphs, and logical organization to form your argument. This should be about 1.5-2 pages long (double-spaced), or about three paragraphs, because you are expected to talk about both articles in detail, plus draw some connectsions.

New York Times article

Article link: "We're all afraid to talk about money. Here's how to break the taboo." (Kristen Wong, *The New York Times*, 28 August 2018) **The NYT paywalled the article, so please use the copy included.**

1. Why is it so hard to talk about money?
2. Why is a class like Personal Finance so important?

Bank of America article

Article link: "10 online and mobile security tips" (n.a., Bank of America, n.d.)

1. So much of this feels like common sense, especially to a generation that may often take the reliance on technology for granted. Do you think reviewing these tips is still important for you, or is this article directed at a different audience? Why?
2. In a 2017 study from Pew Research Center, only 22% of users, regardless of age, regularly update their apps and operating system. (Source) We know why it's important to run system updates, but why might people across the board be so hesitant to do so?

Synthesis

1. In a world that is becoming increasingly more reliant on technology, it's more important than ever to be aware of your own finances and think critically about them. How can we use the digital resources available to us in order to open up conversations about money, so we can become more informed (and help others become more informed) about our finances?