Dynamic Local Average Treatment Effects

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NABE TEC 2024 Session on Causal Machine Learning for Endogeneity Problems in Recommender Systems

I am presenting joint work with Vasilis Syrgkanis.

Paper's QR Code

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Stanford University

Treatment dynamics & noncompliance are rife!

In panel data settings, assignment often depends on prior states, and there is noncompliance.

Example Setting	Education	Marketing	Medical Treatment
Outcome (Y)	Student SAT score	Customer spend	Patient tumor size
Treatment (D)	Enroll in advanced course	Redeem discount	Continue strong drug
Dynamic encouragement (Z)	Offer advanced course if grades were high last year	Offer discount if customer added-to-cart yesterday	Encourage status quo if toxicity is acceptable
Treatment Noncompliance (D≠Z)	Students may not enroll in advanced classes	Customers may not redeem offered discounts	Physician shifts to weaker drug

Are treatment sequence effects identified? E.g. $\mathbb{E}[Y(d) - Y(0,0) \mid D(z) = d, D(0,0) = (0,0)]$

Noncompliance in Dynamic Treatment Regimes

Marketing Example

Each day of Thanksgiving Week, non-members who visited checkout yesterday, did not buy, and visit the checkout page today are offered free membership for the week if they buy.

Noncompliance in Dynamic Treatment Regimes

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Each day of Thanksgiving Week, $\underbrace{\text{non-members}}^{\text{Target population}}$ who visited checkout yesterday, did not buy, and visit the checkout page today are offered free membership for the week if they buy.

Noncompliance in Dynamic Treatment Regimes

Marketing Example

Target population

Purchase Intent

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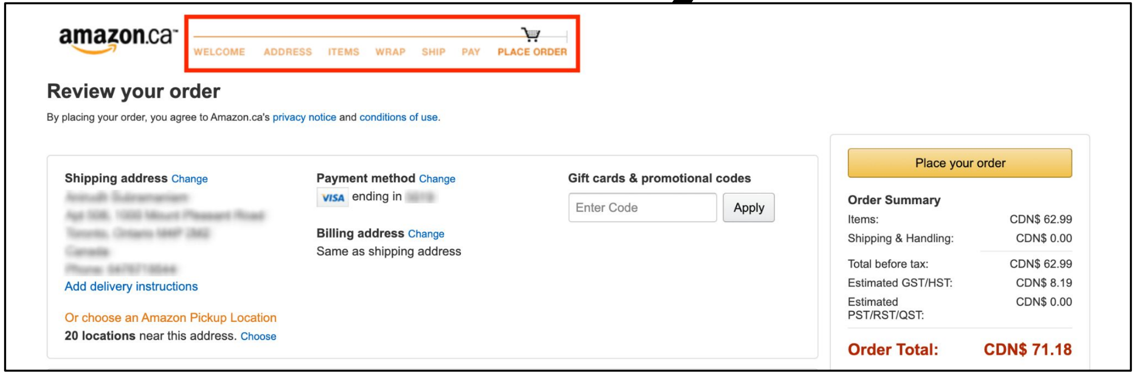

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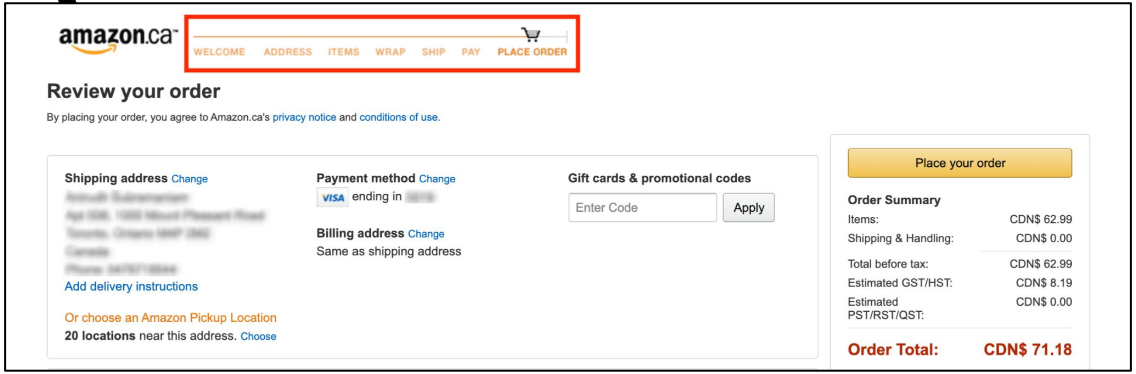

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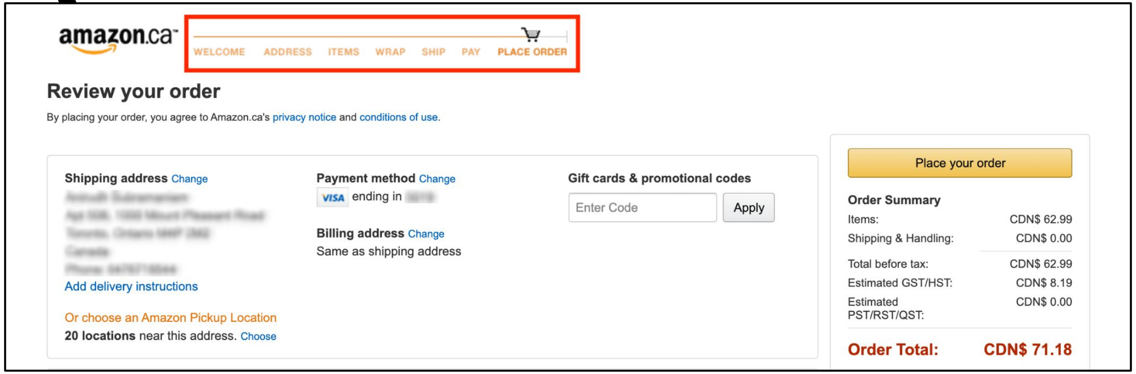

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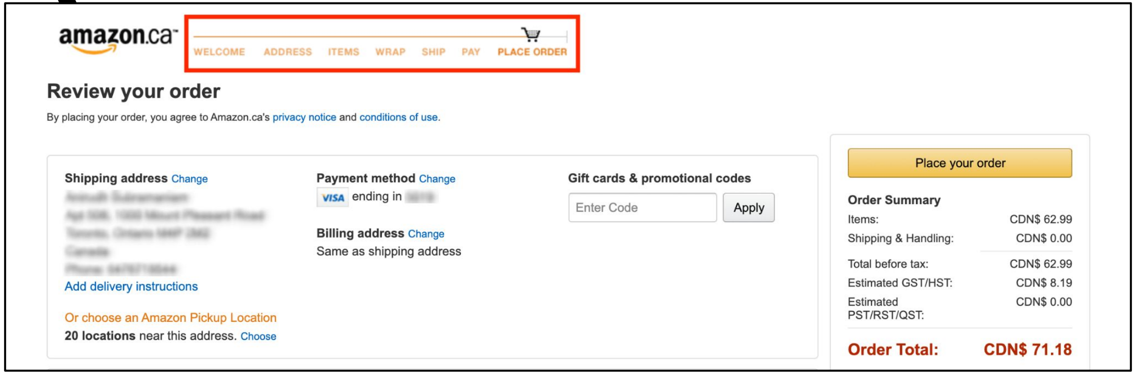
Incentive

Treatment sequence



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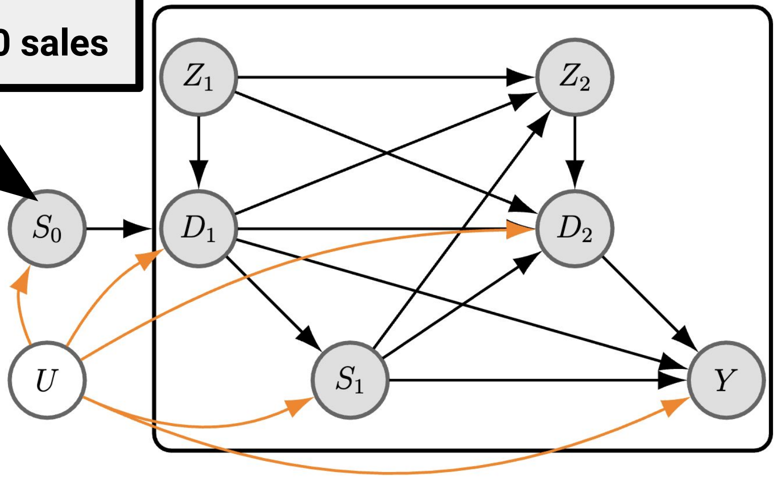
Did not visit checkout or buy, $0 sales



Noncompliance in Dynamic Treatment Regimes

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Not offered free trial at checkout

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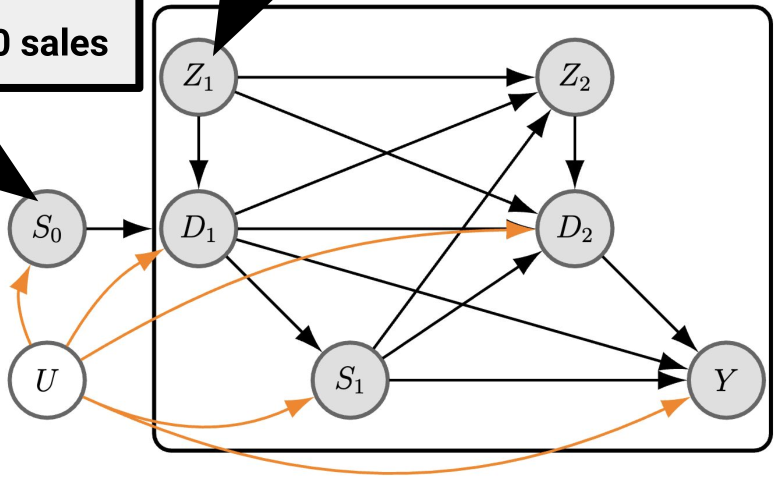

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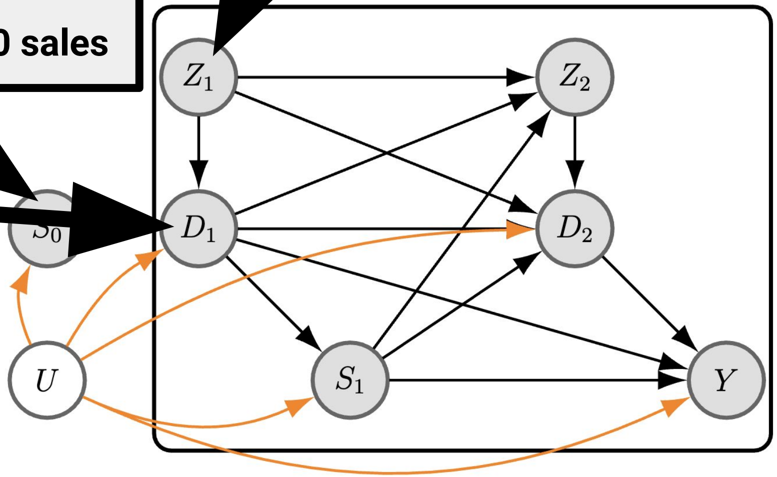
Treatment: D_1 = 0.
Did not redeem at checkout



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State: S_0 = (0, 0, $0).
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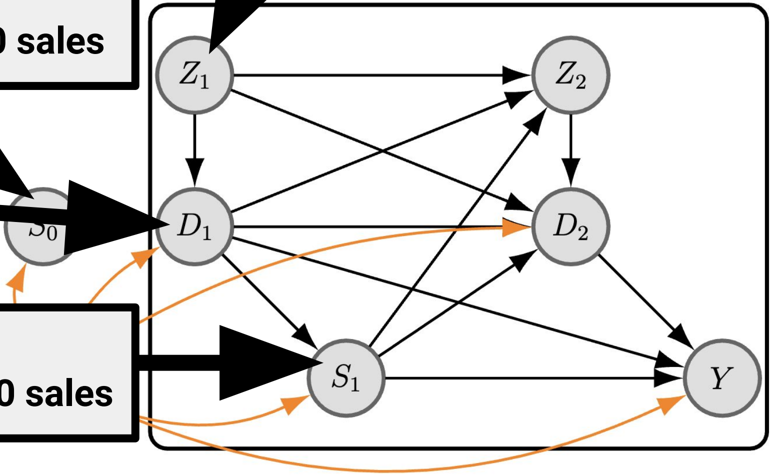
Treatment: D_1 = 0.
Did not redeem at checkout

State: S_1 = (1, 0, $0).
Visited checkout, did not buy, $0 sales



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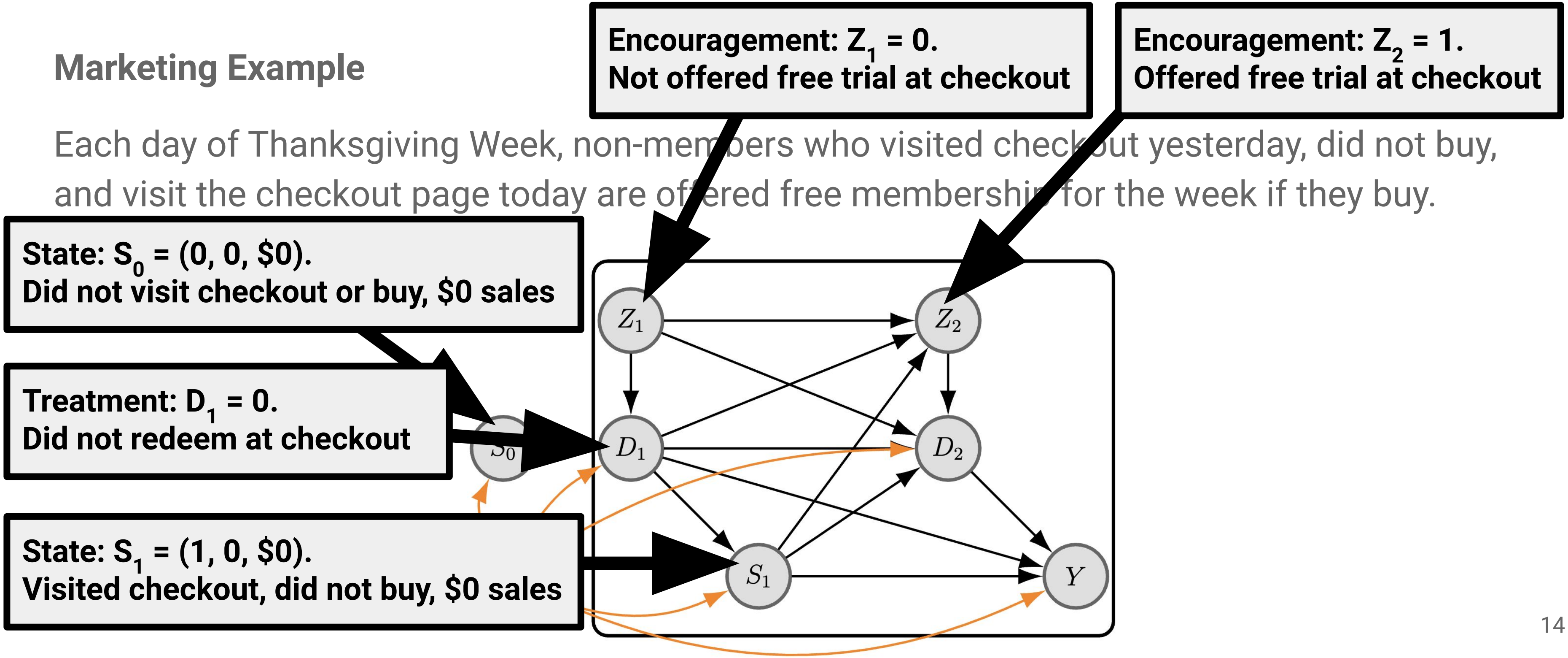

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Encouragement: $Z_1 = 0$.
Not offered free trial at checkout

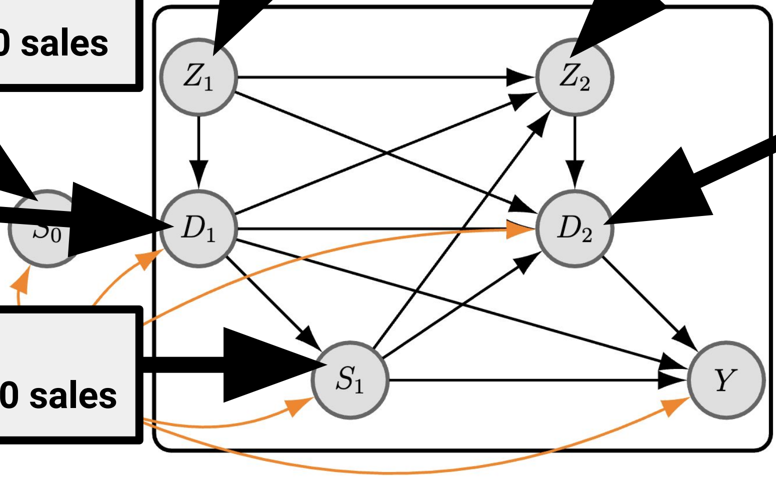
Encouragement: $Z_2 = 1$.
Offered free trial at checkout

State: $S_0 = (0, 0, \$0)$.
Did not visit checkout or buy, \$0 sales

Treatment: $D_2 = 1$.
Redeemed at checkout

Treatment: $D_1 = 0$.
Did not redeem at checkout

State: $S_1 = (1, 0, \$0)$.
Visited checkout, did not buy, \$0 sales



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Offered free trial at checkout

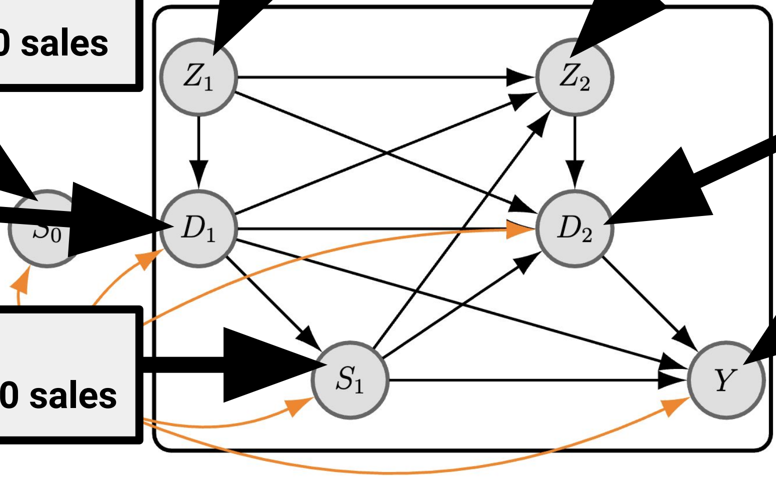
State: S_0 = (0, 0, $0).
Did not visit checkout or buy, $0 sales

Treatment: D_2 = 1.
Redeemed at checkout

Treatment: D_1 = 0.
Did not redeem at checkout

Outcome: Y = $1000.
Holiday season profit

State: S_1 = (1, 0, $0).
Visited checkout, did not buy, $0 sales



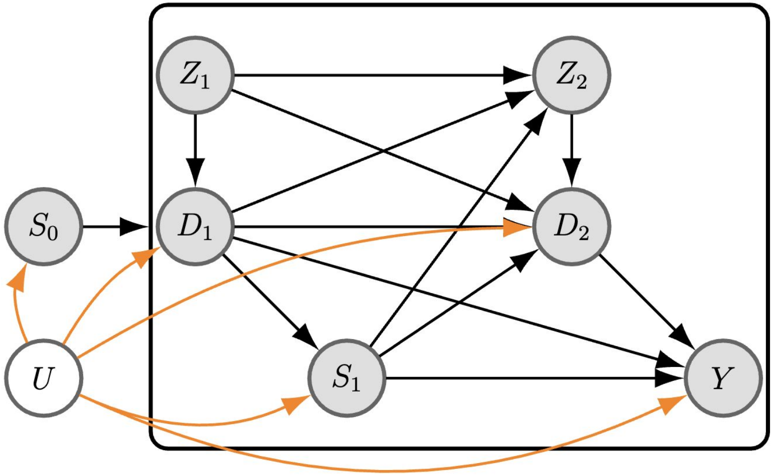
What should we offer non-members next year?

Marketing Example

Each day of Thanksgiving Week, non-members who visited checkout yesterday, did not buy, and visit the checkout page today are offered free membership for the week if they buy.

Question. *Next year*, what should we promote to non-members who transact? On which days?

Perhaps the specific services that were profitable because of the redemption?



Z_t: Offer

D_t: Redeem

S_t: Purchase Intent

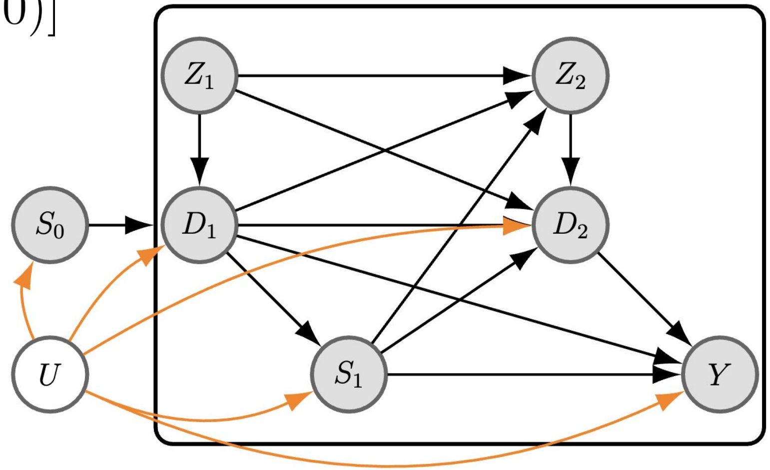
Y: Gross profit

Formally, our estimands are Dynamic LATEs.

Definition of Dynamic Local Average Treatment Effects

Effect of sequence d for those treated with D=d under Z=z & treated with D=(0,0) under Z=(0,0).

$$\mathbb{E}[Y(d) - Y(0,0) \mid D(z) = d, D(0,0) = (0,0)]$$



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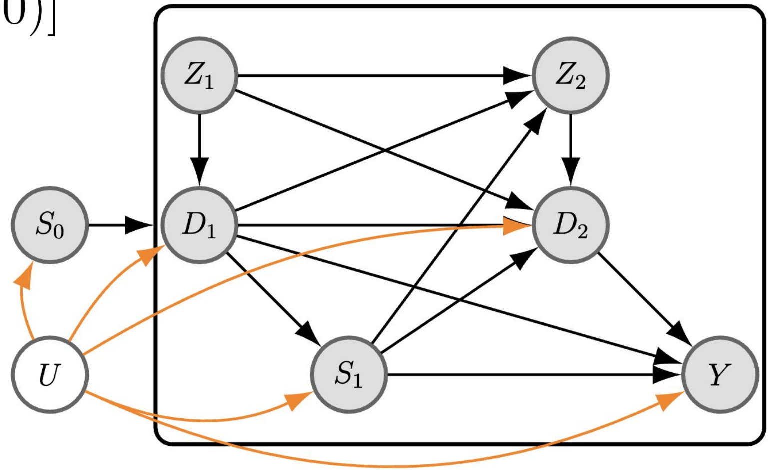
Effect of D on Y for compliers



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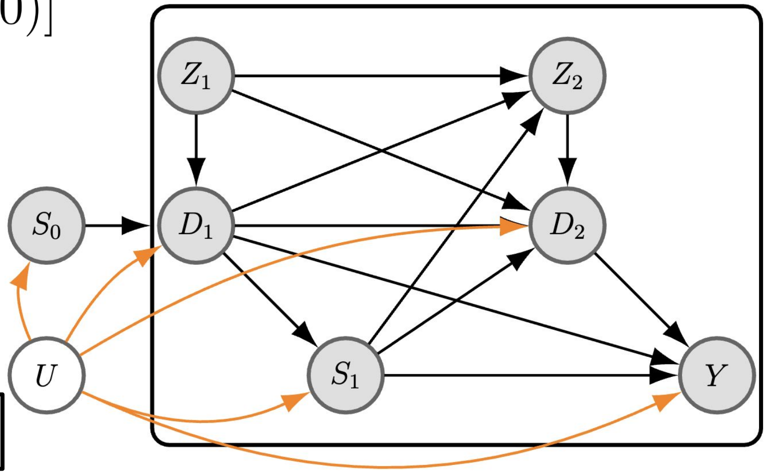
$$\mathbb{E}[Y(d) - Y(0,0) \mid \overbrace{D(z) = d, D(0,0) = (0,0)}^{\textbf{Compliers}}]$$

Effect of D on Y for compliers

Redeem if encouraged

Do not redeem if not encouraged



This is a DTR since offers depend on prior states.

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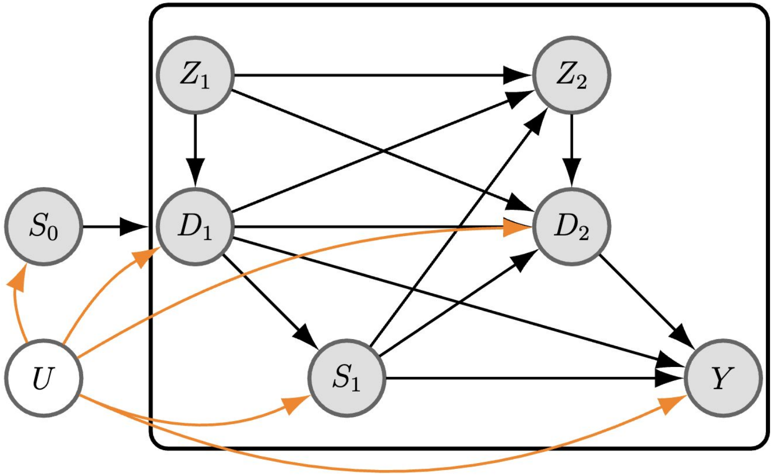

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There is noncompliance since a subset redeems.

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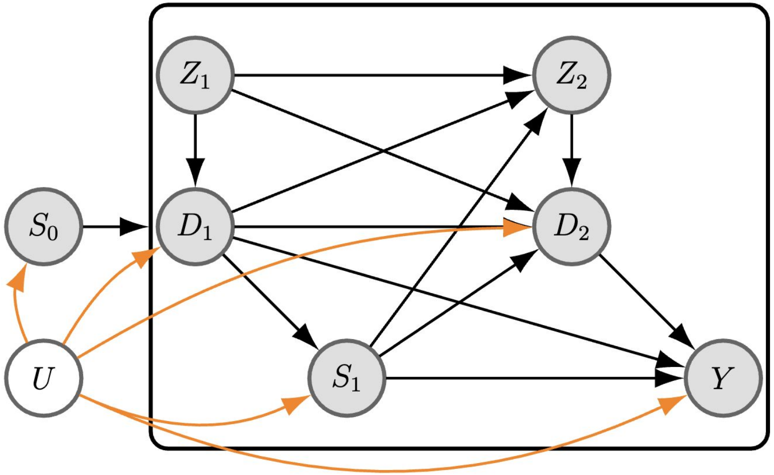

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Treatments, States, & Outcomes are confounded.

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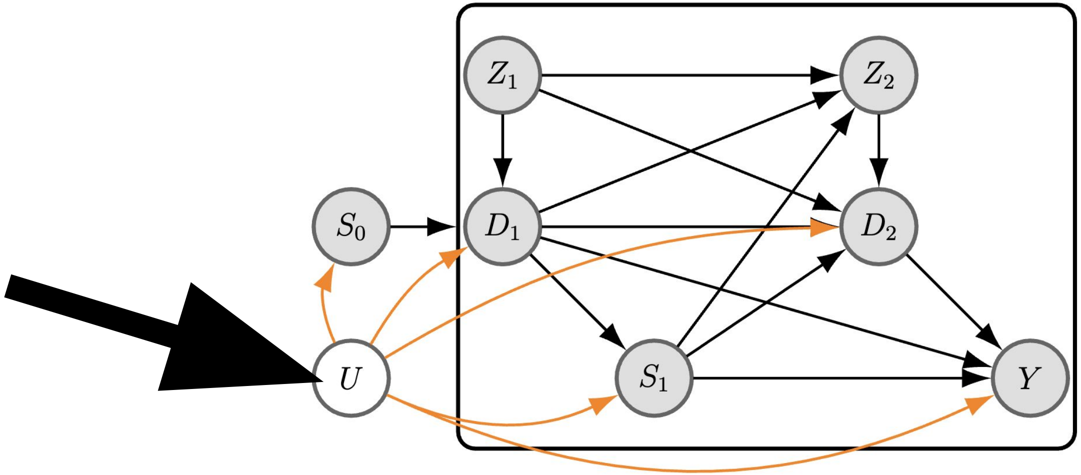

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Imbens & Angrist (‘94) gives ID for static LATEs.

The LATE is identified because it equals an estimable function of the observational distribution.

$$\mathbb{E}[Y(1) - Y(0) \mid D(1) - D(0) = 1] = \frac{\mathbb{E}[Y \mid Z = 1] - \mathbb{E}[Y \mid Z = 0]}{\mathbb{E}[D \mid Z = 1] - \mathbb{E}[D \mid Z = 0]}$$

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Effect of D on Y for compliers

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Effect of Z on Y for everyone

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Effect of D on Y for compliers

Effect of Z on D for everyone

What about the >1 time period, dynamic setting?

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???

1. What about >1 time period setting?
2. Are the assumptions/conditions reasonable?
3. Do the results have the same interpretation?

Thm 1. One Sided Noncompliance enables $\tau_{1,0}, \tau_{0,1}$.

Theorem.

Under sequential extensions of identifying assumptions for LATE and replacing Monotonicity with One Sided Noncompliance, *for **d=z ∈ { (1,0), (0,1) }, we have***

$$\mathbb{E}[Y(d) - Y(0,0) \mid D(z) = d, D(0,0) = (0,0)] = \frac{\mathbb{E}[Y(D(z))] - \mathbb{E}[Y(D(0,0))]}{\Pr\{D(z) = d\}}$$

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Imbens and Angrist (1994)

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1. ***Sequential*** No-interference Consistency for Y, D, S, Z;
2. ***Sequential*** Exclusion Restrictions;
3. ***Sequential*** Ignorability (unconfoundedness);
4. ***Sequential*** Weak Overlap (Positivity; randomization);
5. ***Sequential*** IV Relevance; and
6. Monotonicity and no always-takers (i.e. One Sided Noncompliance).

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Example: Sequential Weak Overlap

$$\Pr\{Z_1 = z_1 \mid S_0\} > 0$$
$$\Pr\{Z_2 = z_2 \mid Z_1 = z_1, D_1, S_0, S_1\} > 0, \forall z \in \mathcal{Z}$$

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May (not) redeem if offered, but cannot redeem otherwise.

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Under sequential extensions of identifying assumptions for LATE and **replacing Monotonicity with One Sided Noncompliance**, *for **d=z ∈ { (1,0), (0,1) }, we have***

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One time period is treated

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Effect of Z on Y for everyone

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Effect of D on Y for compliers

Effect of Z on D for everyone

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A causal effect is identified if it equals an estimable function of the observational data distribution.

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$$\mathbb{E}[Y(D(z))] = \mathbb{E}[\mathbb{E}[\mathbb{E}[Y \mid S, D_1, Z = z] \mid S_0, Z_1 = z_1]]$$
$$\Pr(D(z) = d) = \mathbb{E}[\mathbb{E}[\Pr(D = d \mid S, D_1, Z = z) \mid S_0, Z_1 = z_1]].$$

Thm 2. Cov[$\boldsymbol{\tau}_{i,1,0}$,$D_2(1,1)$ | $D_1(1)$=1,S_0] = 0 gives $\boldsymbol{\tau}_{1,1}$.

Theorem.

Under the assumptions of the previous theorem, if the ***effect*** of not continuing to comply is independent (on average) ***of whether units*** remain compliers (Cov[$\boldsymbol{\tau}_{i,1,0}$,$D_2(1,1)$ | $D_1(1)$=1,S_0] = 0), ***then***

$$\mathbb{E}[Y(1,1) - Y(0,0) \mid D(1,1) = (1,1), D(0,0) = (0,0)]$$

$$= \frac{\mathbb{E}[\Gamma - \tau_{10}(S_0)\,\mathbb{E}[\Pr(D = (1,0) \mid H_2, Z_2 = 1) \mid S_0, Z_1 = 1]]}{\mathbb{E}[\mathbb{E}[\Pr(D = (1,1) \mid H_2, Z_2 = 1] \mid S_0, Z_1 = 1]]}$$

$$\Gamma \triangleq \mathbb{E}[\mathbb{E}[Y \mid H_2, Z_2 = D_1] \mid S_0, Z_1 = 1] - \mathbb{E}[\mathbb{E}[Y \mid H_2, Z_2 = 0] \mid S_0, Z_1 = 0]$$

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Y(1,0) - Y(0,0)

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Effect of D on Y for compliers

$$= \frac{\mathbb{E}[\Gamma - \tau_{10}(S_0)\,\mathbb{E}[\Pr(D = (1,0) \mid H_2, Z_2 = 1) \mid S_0, Z_1 = 1]]}{\mathbb{E}[\mathbb{E}[\Pr(D = (1,1) \mid H_2, Z_2 = 1] \mid S_0, Z_1 = 1]]}$$

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Under the assumptions of the previous theorem, if the ***effect*** of not continuing to comply is

Effect of adaptive (wrt D) policy Z on Y for everyone

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$$\mathbb{E}[Y(1,1) - Y(0,0) \mid D(1,1) = (1,1), D(0,0) = (0,0)]$$

Effect of D on Y for compliers

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Correction term

Effect of D on Y for compliers

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Effect of Z on D for everyone

Examples 1,2: Staggered Adoption & Compliance

Theorem.

Under the assumptions of the first theorem, if Staggered Adoption Pr[D_2=1 | S_0, Z_1=1, D_1=1] = 1 or “Staggered Compliance” Pr[D_2(1,1)=1 | S_0, Z_1=1 D_1=1] = 1 holds, ***then***

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Examples 1,2: Staggered Adoption & Compliance

Once treated, remain treated

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No correction term!

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Ratio of effects (for everyone) of (adaptive Z) on Y, D

Examples 3+: Put structure on confounders.

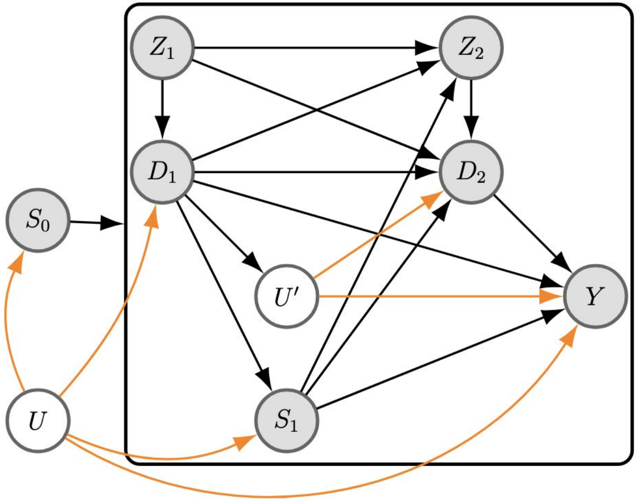

Figure 3: Example Directed Acyclic Graphs that satisfies assumption when combined with $Y(1,0) - Y(0,0) \perp\!\!\!\perp U'$. For example, $f_Y(d, s, u, u', \epsilon) = \tau_{10}(s, u, \epsilon)d_1 + \tau_{01}(s, u, u', \epsilon)d_2 + \tau_{11}(s, u, u', \epsilon)d_1 d_2 + Y_{0,0}(s, u, u', \epsilon)$.

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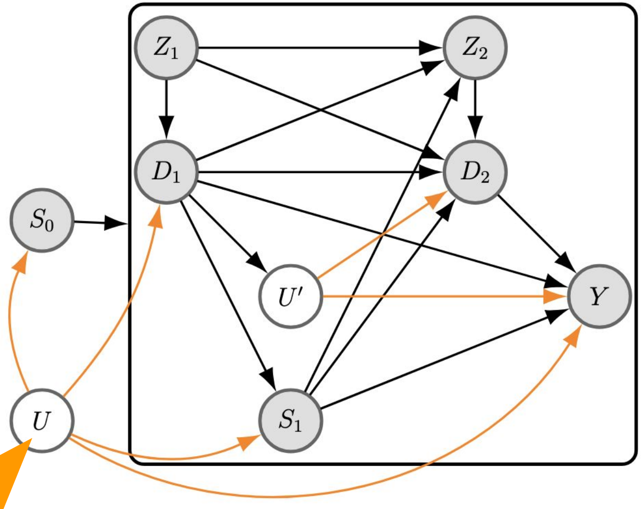

assumption when combined with $Y(1,0)-Y(0,0) \perp$

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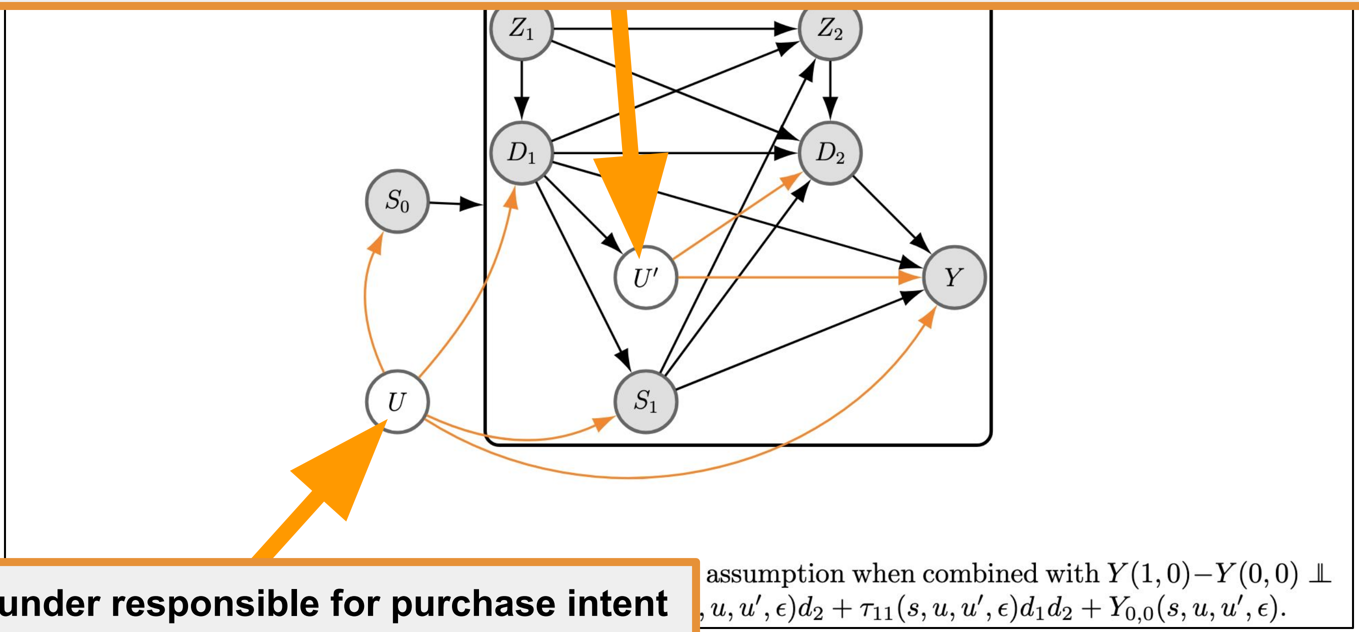
E.g. confounder responsible for purchase intent

Examples 3+: Put structure on confounders.

E.g. confounder responsible for subsequent platform side changes (e.g. free trial is extended)



E.g. confounder responsible for purchase intent

Check out our paper for more results!

Formally, we show causal quantities below equal estimable functions of observational data.

Local Intent to Treat: $E[Y(D(\mathbf{z})) - Y(\mathbf{0}) \mid D(\mathbf{z}) \neq \mathbf{0}]$ for every z not equal to **0**.

Dynamic LATE: $E[Y(\mathbf{d}) - Y(\mathbf{0}) \mid D(\mathbf{z}) = \mathbf{d}]$.

- When-to-Treat: **d**=**z** and **z**,**d** are standard basis vectors.
- Always-Treat: **d**=**z** and **z**,**d** equal vectors of ones.
- When-to-Start: **d**=**z**, **z** non-decreasing in its coordinates, and one in **z**'s last (T'th) entry.
- When-to-Comply: $\mathbf{d}_{\geq t'}=\mathbf{z}_{\geq t'}$ for some t' and $d_t=0$ for $t<t'$.

We can also identify all of these conditional on baseline covariates (Heterogeneous LATEs).

We also provide estimation, inference, generalization to T time periods, and simulations.

Treatment dynamics & noncompliance are rife!

In panel data settings, assignment often depends on prior states, and there is noncompliance.

Example Setting	**Education**	**Marketing**	**Medical Treatment**
Outcome (Y)	Student SAT score	Customer spend	Patient tumor size
Treatment (D)	Enroll in advanced course	Redeem discount	Continue strong drug
Dynamic encouragement (Z)	Offer advanced course if grades were high last year	Offer discount if customer added-to-cart yesterday	Encourage status quo if toxicity is acceptable
Treatment Noncompliance (D≠Z)	Students may not enroll in advanced classes	Customers may not redeem offered discounts	Physician shifts to weaker drug

Thank you!

Dynamic Local Average Treatment Effects

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