



Market Measure *m* of an Arm's Length Result

Addressing Discount Rates Controversies

Kroll Team

DCF Measures in CSA

$$x_0^{\text{LIC}} = \frac{\mathbb{E}_P(E_1^{\text{LIC}})}{1 + r^{\text{LIC}}} + \frac{\mathbb{E}_P(E_2^{\text{LIC}})}{(1 + r^{\text{LIC}})^2} + \cdots + \frac{\mathbb{E}_P(E_T^{\text{LIC}})}{(1 + r^{\text{LIC}})^T}$$

$$x_0^{\text{CSA}} = \frac{\mathbb{E}_P(E_1^{\text{CSA}})}{1 + r^{\text{CSA}}} + \frac{\mathbb{E}_P(E_2^{\text{CSA}})}{(1 + r^{\text{CSA}})^2} + \cdots + \frac{\mathbb{E}_P(E_T^{\text{CSA}})}{(1 + r^{\text{CSA}})^T}$$

RAP applies to **asset prices** $x_0^{\text{LIC}}, x_0^{\text{CSA}}$ such that

$$x_0^{\text{CSA}} - \text{PCT} = x_0^{\text{LIC}}$$

Market Measures in CSA

$$x_0^{\text{LIC}} = \left(\frac{\mathbb{E}_P(E_1^{\text{LIC}})}{1 + r_1^{\text{LIC}}} = e_{0,1}^{\text{LIC}} \right) + \left(\frac{\mathbb{E}_P(E_2^{\text{LIC}})}{(1 + r_2^{\text{LIC}})^2} = e_{0,2}^{\text{LIC}} \right) + \cdots + \left(\frac{\mathbb{E}_P(E_T^{\text{LIC}})}{(1 + r_T^{\text{LIC}})^T} = e_{0,T}^{\text{LIC}} \right)$$

$$x_0^{\text{CSA}} = \left(\frac{\mathbb{E}_P(E_1^{\text{CSA}})}{1 + r_1^{\text{CSA}}} = e_{0,1}^{\text{CSA}} \right) + \left(\frac{\mathbb{E}_P(E_2^{\text{CSA}})}{(1 + r_2^{\text{CSA}})^2} = e_{0,2}^{\text{CSA}} \right) + \cdots + \left(\frac{\mathbb{E}_P(E_T^{\text{CSA}})}{(1 + r_T^{\text{CSA}})^T} = e_{0,T}^{\text{CSA}} \right)$$

RAP applies to **earnings prices** $\{e_{0,t}^{\text{LIC}}\}_t$, $\{e_{0,t}^{\text{CSA}}\}_t$ for all t such that



$$e_0^{\text{LIC}} = e_{0,t}^{\text{LIC}}$$

$$e_0^{\text{CSA}} = e_{0,t}^{\text{CSA}}$$

DIFFERENCE

- The RAP of the regulation applies to **asset prices** but not to asset earnings
 - *“How much would the **asset** cost in the financial markets, if traded?”*
- The RAP of the market measure applies to **each earnings** constituent of each asset
 - *“How much would a claim to each **earnings** of an asset cost in the financial markets, if traded?”*
- The RAP applied to earnings satisfies the RAP applied to asset prices, it is a more stringent requirement that takes advantage of the current known price of earnings; the current price of assets is not known!

IMPORTANT TECHNICAL POINT

- The DCF discount rates are obtained independently from the subjective financial projection of a taxpayer. This produces a lot of controversies
- The market measure of earnings discounts are a function of the subjective financial projection of a taxpayer. **This makes the subjective financial projection of a taxpayer irrelevant** (periodic adjustment regulation moot)
- Forcing earnings to be fairly priced (traded in a competitive market) ensures that only market considerations translate into asset prices, no subjectivity is left

$$r_t^{\text{LIC/CSA}} = f(\mathbb{E}_P(E_t^{\text{LIC/CSA}}), e_o^{\text{LIC/CSA}}), t = 1, \dots, T$$


THIS IS KEY TO ELIMINATE CONTROVERSY

BECAUSE $e_o^{\text{LIC/CSA}}$ IS FIXED

DCF Discount Solution

- Once the process $\{r_t^{\text{LIC/CSA}}\}_t$ (in green now) is calculated from the financial projection solve for the DCF discount rate $r^{\text{LIC/CSA}}$:

$$\begin{aligned} x_0^{\text{LIC/CSA}} &= \left(\frac{\mathbb{E}_P(E_1^{\text{LIC/CSA}})}{1 + r_1^{\text{LIC/CSA}}} = e_0^{\text{LIC/CSA}} \right) + \left(\frac{\mathbb{E}_P(E_2^{\text{LIC/CSA}})}{(1 + r_2^{\text{LIC/CSA}})^2} = e_0^{\text{LIC/CSA}} \right) + \dots \\ &+ \left(\frac{\mathbb{E}_P(E_T^{\text{LIC/CSA}})}{(1 + r_T^{\text{LIC/CSA}})^T} = e_0^{\text{LIC/CSA}} \right) = \frac{\mathbb{E}_P(E_1^{\text{LIC/CSA}})}{1 + r^{\text{LIC/CSA}}} + \frac{\mathbb{E}_P(E_2^{\text{LIC/CSA}})}{(1 + r^{\text{LIC/CSA}})^2} + \dots + \frac{\mathbb{E}_P(E_T^{\text{LIC/CSA}})}{(1 + r^{\text{LIC/CSA}})^T} \end{aligned}$$

- Suppose the financial projection is inflated by 100 percent, 200 percent, etc.: $x_0^{\text{LIC/CSA}}$ will **NOT CHANGE!** **The subjectivity of a financial projection is eliminated from the equation**

APPLICATION TO A COST SHARING ARRANGEMENT

THE CSA

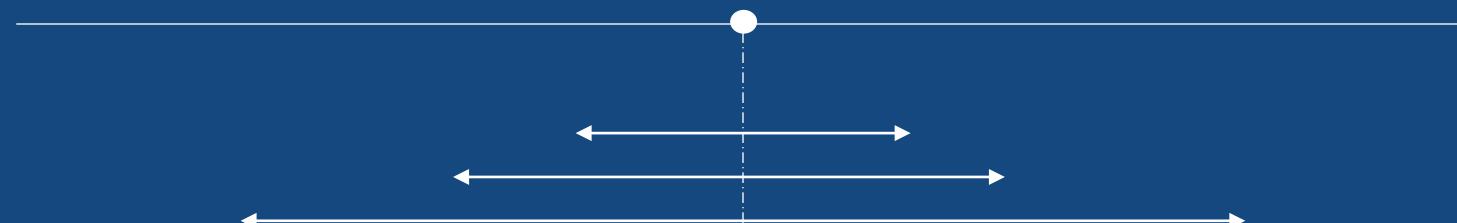
- Inbound CSA into the United States
- Taxpayer developed a subjective financial projection for four years with terminal values as the CSA is indefinite
- The financial projection was provided as a process $\{\mathbb{E}_p(E_t)\}_t$ that conceals (1) **the probability measure P** , and (2) **the standard-deviation of the earnings around their mean, which is what we price!** **HOW CAN WE PRICE SOMETHING WE HAVE NO MEASURE OF?** This is the fundamental reason we have controversy.

Value =

EXPECTATION OF UNCERTAIN CASH FLOW (*probability-weighted average cash flow*)

1+DISCOUNT RATE (*price standard-deviation cash flow*)

Expectation (probability-weighted average)



Three Different Volatilities (standard-deviation) concealed in the SAME expectation
WHICH ONE IS THE ONE FOR THE TAXPAYER'S FINANCIAL PROJECTION????

THE CSA (continued)

- Assume the *risk-free rate process* $\{r_t\}_t = \{2\%, 3\%, 4\%, 5\%, 6\%, 6\%, 6\%, \dots\}$
- The risk-free rate process is quoted on an annualized, continuously compounded basis. It is the zero-coupon U.S. Treasury yield curve.
- That yield curve is known at the date $t = 0$ of the CSA
- Note that none of what we are about to do requires functional information; the financial markets only care about the expectation (level) and the volatility (risk) of a cash flow. Risk-aversion is “risk of deviation from expectation;” that is WHAT WE PRICE!
- The date $t = 0$ *pro forma* earnings of the divisional interest contemplated in the CSA are $e_o = \$100,000,000$ in the licensing alternative and $e_o = \$110,000,000$ in the CSA alternative, both known at date $t = 0$ and both measuring the price of these earnings

STEP 1: Calculate the Process $\{r_t^X\}_t$ Licensing Alternative

Dates	Financial Projection	Martingale Pricing	Discount Rate	Risk-Free Component	Price of Risk
$t = 0$	$e_0 = \$100,000,000$		-	-	-
$t = 1$	$\$105,000,000$	$\$100,000,000$	4.88%	2%	2.88%
$t = 2$	$\$125,000,000$	$\$100,000,000$	11.16%	3%	8.16%
$t = 3$	$\$130,000,000$	$\$100,000,000$	8.75%	4%	4.75%
$t = 4$	$\$150,000,000$	$\$100,000,000$	10.14%	5%	5.14%
Terminal	$\$3,844,188,628$	$\$2,222,222,222$	10.96%	6%	4.96%
Value Licensing Alternative:		$\$2,622,222,222$			

STEP 2: Calculate the DCF DR r_L Licensing Alternative

$$\frac{\$2,622,222,222}{\$105,000,000} + \frac{\$125,000,000}{(1 + r_L)^2} + \frac{\$130,000,000}{(1 + r_L)^3} + \frac{\$150,000,000}{(1 + r_L)^4} + \frac{\$3,844,188,628}{(1 + r_L)^5}$$

The solution is $r_L = 11.44$ percent

- At that DCF discount rate, *the price of each future earnings is unfair*, there exist arbitrage trades, but the price of the asset (value of licensing alternative) is fair and satisfies the RAP
- Once again, IT IS NOT AN ISSUE that the price of the earnings are arbitrageable, and it is **not a criticism of DCF**

STEP 3: Calculate the Process $\{r_t^X\}_t$ CSA Alternative

Dates	Financial Projection	Martingale Pricing	Discount Rate	Risk-Free Component	Price of Risk
$t = 0$	$e_0 = \$110,000,000$	-	-	-	-
$t = 1$	$\$120,000,000$	$\$110,000,000$	8.70%	2%	6.70%
$t = 2$	$\$145,000,000$	$\$110,000,000$	13.81%	3%	10.81%
$t = 3$	$\$165,000,000$	$\$110,000,000$	13.51%	4%	9.51%
$t = 4$	$\$185,000,000$	$\$110,000,000$	13.00%	5%	8.00%
Terminal	$\$5,032,024,157$	$\$2,444,444,444$	14.44%	6%	14.44%
Value Licensing Alternative:		$\$2,884,444,444$			

STEP 4: Calculate the DCF DR r_{CSA} CSA Alternative

$$\frac{\$2,884,444,444}{\$120,000,000} + \frac{\$145,000,000}{(1 + r_{CSA})^2} + \frac{\$165,000,000}{(1 + r_{CSA})^3} + \frac{\$185,000,000}{(1 + r_{CSA})^4} + \frac{\$5,032,024,157}{(1 + r_{CSA})^5}$$

The solution is $r_{CSA} = 15.39$ percent

- At that DCF discount rate, *the price of each future earnings is unfair*, there is an arbitrage trade, but the price of the asset (value of CSA alternative) is fair and satisfies the RAP
- Once again, IT IS NOT AN ISSUE that the price of the earnings are arbitrageable, and it is (still) **not a criticism of DCF**

STEP 5: Calculate the PCT

PCT

= \$2,884,444,444

- \$2,622,222,222 =

= \$262,222,222

- You get that measure with a DCF at the discount rates of 11.44 percent for the licensing alternative and 15.39 percent for the cost sharing alternative, GIVEN THE FINANCIAL PROJECTION OF THE TAXPAYER!
- Change the financial projection of the taxpayer and the discount rates that correctly yield the same PCT change. The PCT SHOULD NOT CHANGE BASED ON SUBJECTIVE PROJECTIONS

How Arbitrageable are Earnings Prices under DCF?

Illustration with Licensing Alternative

Dates	Financial Projection	Martingale Prices	DCF Prices	Value of Arbitrage
$t = 0$	$e_0 = \$100,000$	-	-	
$t = 1$	\$105,000,000	\$100,000,000 (short)	\$94,979,647 (long)	\$5,020,353
$t = 2$	\$125,000,000	\$100,000,000 (long)	\$102,280,423 (short)	\$2,283,423
$t = 3$	\$130,000,000	\$100,000,000 (short)	\$96,220,390 (long)	\$3,779,610
$t = 4$	\$150,000,000	\$100,000,000 (long)	\$100,428,337 (short)	\$428,337
Terminal	\$3,844,188,628	\$2,222,222,222 (long)	\$2,328,149,985 (short)	\$105,927,763
Value Licensing Alternative:		\$2,622,222,222 (not arbitrageable under DCF or market measure)		

CONCLUSION

- A market measures applies the RAP to the **discounted earnings price process** that confers the asset its value. Satisfying that RAP satisfies the regulatory RAP
- It is **THE LOWEST POSSIBLE MEASURE OF AN ARM'S LENGTH RESULT AND IS THE SAME AS A CORRECT DCF ONE** (both reflect competitive financial trading of the **asset**)
- *Calculating the PCT under a market measure does not require discount rates or a financial projection, all is needed is the price of the earnings for the year preceding valuation date; these are always available pro forma*
- *Getting that measure of the PCT allows to precisely calculate the discount rates at which to discount earnings in an application of DCF*
- *TRYING TO GET THE 11.44 PERCENT AND 15.39 PERCENT DISCOUNT RATES THAT YIELD THE CORRECT RAP PRICE x_0 OF THE ASSETS USING COMPARABLES AND CAPM, SOLELY BASED ON QUALITATIVE INFORMATION ABOUT CASH FLOW RISK, IS WHAT IS TYPICALLY DONE AND LEADS TO ENDLESS CONTROVERSIES*
- *ELIMINATING CONTROVERSY*: taxpayers can provide any subjective financial projection, reasonable or not, *IT DOES NOT MATTER*. A market measure does not depend on these projections, since the DCF discount rates will automatically correct for that. *This is how to eliminate these endless controversies.*



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Thank You





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