



Market Measure *m* of an Arm's Length Result

Addressing Discount Rates Controversies

Kroll Team

DCF Measures in CSA

$$x_0^{\text{LIC}} = \frac{\mathbb{E}_P(E_1^{\text{LIC}})}{1 + r^{\text{LIC}}} + \frac{\mathbb{E}_P(E_2^{\text{LIC}})}{(1 + r^{\text{LIC}})^2} + \dots + \frac{\mathbb{E}_P(E_T^{\text{LIC}})}{(1 + r^{\text{LIC}})^T}$$

$$x_0^{\text{CSA}} = \frac{\mathbb{E}_P(E_1^{\text{CSA}})}{1 + r^{\text{CSA}}} + \frac{\mathbb{E}_P(E_2^{\text{CSA}})}{(1 + r^{\text{CSA}})^2} + \dots + \frac{\mathbb{E}_P(E_T^{\text{CSA}})}{(1 + r^{\text{CSA}})^T}$$

RAP applies to **asset prices** x_0^{LIC} , x_0^{CSA} such that

$$x_0^{\text{CSA}} - \text{PCT} = x_0^{\text{LIC}}$$

Market Measures in CSA

$$x_0^{\text{LIC}} = \left(\frac{\mathbb{E}_P(E_1^{\text{LIC}})}{1 + r_1^{\text{LIC}}} = e_{0,1}^{\text{LIC}} \right) + \left(\frac{\mathbb{E}_P(E_2^{\text{LIC}})}{(1 + r_2^{\text{LIC}})^2} = e_{0,2}^{\text{LIC}} \right) + \dots + \left(\frac{\mathbb{E}_P(E_T^{\text{LIC}})}{(1 + r_T^{\text{LIC}})^T} = e_{0,T}^{\text{LIC}} \right)$$

$$x_0^{\text{CSA}} = \left(\frac{\mathbb{E}_P(E_1^{\text{CSA}})}{1 + r_1^{\text{CSA}}} = e_{0,1}^{\text{CSA}} \right) + \left(\frac{\mathbb{E}_P(E_2^{\text{CSA}})}{(1 + r_2^{\text{CSA}})^2} = e_{0,2}^{\text{CSA}} \right) + \dots + \left(\frac{\mathbb{E}_P(E_T^{\text{CSA}})}{(1 + r_T^{\text{CSA}})^T} = e_{0,T}^{\text{CSA}} \right)$$

RAP applies to earnings prices $\{e_{0,t}^{\text{LIC}}\}_t, \{e_{0,t}^{\text{CSA}}\}_t$ for all t such that

$$e_0^{\text{LIC}} = e_{0,t}^{\text{LIC}}$$

$$e_0^{\text{CSA}} = e_{0,t}^{\text{CSA}}$$



DIFFERENCE

- The RAP of the regulation applies to **asset prices** but not to asset earnings
 - *“How much would the **asset** cost in the financial markets, if traded?”*
- The RAP of the market measure applies to **each earnings** constituent of each asset
 - *“How much would a claim to each **earnings** of an asset cost in the financial markets, if traded?”*
- The RAP applied to earnings satisfies the RAP applied to asset prices, it is a more stringent requirement that takes advantage of the current known price of earnings; the current price of assets is not known!

IMPORTANT TECHNICAL POINT

- The DCF discount rates are obtained independently from the subjective financial projection of a taxpayer. This produces a lot of controversies
- The market measure of earnings discounts are a function of the subjective financial projection of a taxpayer. **This makes the subjective financial projection of a taxpayer irrelevant** (periodic adjustment regulation moot)
- Forcing earnings to be fairly priced (traded in a competitive market) ensures that only market considerations translate into asset prices, no subjectivity is left

$$r_t^{\text{LIC/CSA}} = f(\mathbb{E}_P(E_t^{\text{LIC/CSA}}), e_0^{\text{LIC/CSA}}), t = 1, \dots, T$$


THIS IS KEY TO ELIMINATE CONTROVERSY

BECAUSE $e_0^{\text{LIC/CSA}}$ IS FIXED

DCF Discount Solution

- Once the process $\{r_t^{\text{LIC/CSA}}\}_t$ (in *green* now) is calculated from the financial projection solve for the DCF discount rate $r^{\text{LIC/CSA}}$;

$$\begin{aligned}
 x_0^{\text{LIC/CSA}} &= \left(\frac{\mathbb{E}_P(E_1^{\text{LIC/CSA}})}{1 + r_1^{\text{LIC/CSA}}} = e_0^{\text{LIC/CSA}} \right) + \left(\frac{\mathbb{E}_P(E_2^{\text{LIC/CSA}})}{(1 + r_2^{\text{LIC/CSA}})^2} = e_0^{\text{LIC/CSA}} \right) + \dots \\
 &+ \left(\frac{\mathbb{E}_P(E_T^{\text{LIC/CSA}})}{(1 + r_T^{\text{LIC/CSA}})^T} = e_0^{\text{LIC/CSA}} \right) = \frac{\mathbb{E}_P(E_1^{\text{LIC/CSA}})}{1 + r^{\text{LIC/CSA}}} + \frac{\mathbb{E}_P(E_2^{\text{LIC/CSA}})}{(1 + r^{\text{LIC/CSA}})^2} + \dots + \frac{\mathbb{E}_P(E_T^{\text{LIC/CSA}})}{(1 + r^{\text{LIC/CSA}})^T}
 \end{aligned}$$

- Suppose the financial projection is inflated by 100 percent, 200 percent, etc.: $x_0^{\text{LIC/CSA}}$ will **NOT CHANGE!** The subjectivity of a financial projection is eliminated from the equation

APPLICATION TO A COST SHARING ARRANGEMENT

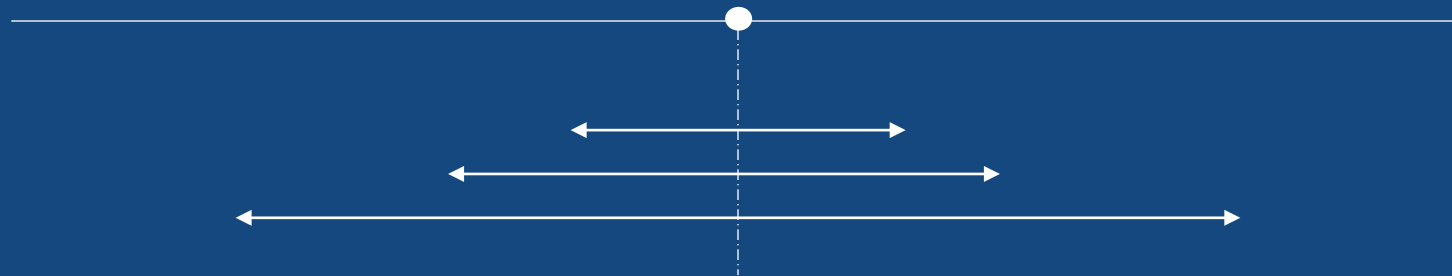
THE CSA

- Inbound CSA into the United States
- Taxpayer developed a subjective financial projection for four years with terminal values as the CSA is indefinite
- The financial projection was provided as a process $\{\mathbb{E}_p(E_t)\}_t$ that conceals (1) **the probability measure P** , and (2) **the standard-deviation of the earnings around their mean, which is what we price!** **HOW CAN WE PRICE SOMETHING WE HAVE NO MEASURE OF?** This is the fundamental reason we have controversy.

Value =

$$\frac{\text{EXPECTATION OF UNCERTAIN CASH FLOW (probability-weighted average cash flow)}}{1 + \text{DISCOUNT RATE (price standard-deviation cash flow)}}$$

Expectation (probability-weighted average)



Three Different Volatilities (standard-deviation) concealed in the SAME expectation
WHICH ONE IS THE ONE FOR THE TAXPAYER'S FINANCIAL PROJECTION????

THE CSA (continued)

- Assume the risk-free rate process $\{r_t\}_t = \{2\%, 3\%, 4\%, 5\%, 6\%, 6\%, 6\%, \dots\}$
- The risk-free rate process is quoted on an annualized, continuously compounded basis. It is the zero-coupon U.S. Treasury yield curve.
- That yield curve is known at the date $t = 0$ of the CSA
- Note that none of what we are about to do requires functional information; the financial markets only care about the expectation (level) and the volatility (risk) of a cash flow. Risk-aversion is “risk of deviation from expectation;” that is WHAT WE PRICE!
- The date $t = 0$ *pro forma* earnings of the divisional interest contemplated in the CSA are $e_0 = \$100,000,000$ in the licensing alternative and $e_0 = \$110,000,000$ in the CSA alternative, both known at date $t = 0$ and both measuring the price of these earnings

STEP 1: Calculate the Process $\{r_t^X\}_t$

Licensing Alternative

Dates	Financial Projection	Martingale Pricing	Discount Rate	Risk-Free Component	Price of Risk
$t = 0$	$e_0 = \$100,000,000$		-	-	-
$t = 1$	\$105,000,000	\$100,000,000	4.88%	2%	2.88%
$t = 2$	\$125,000,000	\$100,000,000	11.16%	3%	8.16%
$t = 3$	\$130,000,000	\$100,000,000	8.75%	4%	4.75%
$t = 4$	\$150,000,000	\$100,000,000	10.14%	5%	5.14%
Terminal	\$3,844,188,628	\$2,222,222,222	10.96%	6%	4.96%
Value Licensing Alternative:		\$2,622,222,222			

STEP 2: Calculate the DCF DR r_L Licensing Alternative

$$= \frac{\$2,622,222,222}{1 + r_L} + \frac{\$105,000,000}{(1 + r_L)^2} + \frac{\$125,000,000}{(1 + r_L)^3} + \frac{\$130,000,000}{(1 + r_L)^4} + \frac{\$150,000,000}{(1 + r_L)^5} + \frac{\$3,844,188,628}{(1 + r_L)^5}$$

The solution is $r_L = 11.44$ percent

- At that DCF discount rate, *the price of each future earnings is unfair*, there exist arbitrage trades, but the price of the asset (value of licensing alternative) is fair and satisfies the RAP
- Once again, IT IS NOT AN ISSUE that the price of the earnings are arbitrageable, and it is **not a criticism of DCF**

STEP 3: Calculate the Process $\{r_t^X\}_t$

CSA Alternative

Dates	Financial Projection	Martingale Pricing	Discount Rate	Risk-Free Component	Price of Risk
$t = 0$	$e_0 = \$110,000,000$	-	-	-	-
$t = 1$	\$120,000,000	\$110,000,000	8.70%	2%	6.70%
$t = 2$	\$145,000,000	\$110,000,000	13.81%	3%	10.81%
$t = 3$	\$165,000,000	\$110,000,000	13.51%	4%	9.51%
$t = 4$	\$185,000,000	\$110,000,000	13.00%	5%	8.00%
Terminal	\$5,032,024,157	\$2,444,444,444	14.44%	6%	14.44%
Value Licensing Alternative:		\$2,884,444,444			

STEP 4: Calculate the DCF DR r_{CSA} CSA Alternative

$$\begin{aligned} & \$2,884,444,444 \\ &= \frac{\$120,000,000}{1 + r_{CSA}} + \frac{\$145,000,000}{(1 + r_{CSA})^2} + \frac{\$165,000,000}{(1 + r_{CSA})^3} + \frac{\$185,000,000}{(1 + r_{CSA})^4} + \frac{\$5,032,024,157}{(1 + r_{CSA})^5} \end{aligned}$$

The solution is $r_{CSA} = 15.39$ percent

- At that DCF discount rate, *the price of each future earnings is unfair*, there is an arbitrage trade, but the price of the asset (value of CSA alternative) is fair and satisfies the RAP
- Once again, IT IS NOT AN ISSUE that the price of the earnings are arbitrageable, and it is (still) **not a criticism of DCF**

STEP 5: Calculate the PCT

PCT

= \$2, 884, 444, 444

– \$2, 622, 222, 222 =

= \$262, 222, 222

- You get that measure with a DCF at the discount rates of 11.44 percent for the licensing alternative and 15.39 percent for the cost sharing alternative, GIVEN THE FINANCIAL PROJECTION OF THE TAXPAYER!
- Change the financial projection of the taxpayer and the discount rates that correctly yield the same PCT change. The PCT SHOULD NOT CHANGE BASED ON SUBJECTIVE PROJECTIONS

How **Arbitrageable** are Earnings Prices under DCF?

Illustration with Licensing Alternative

Dates	Financial Projection	Martingale Prices	DCF Prices	Value of Arbitrage
$t = 0$	$e_0 = \$100,000$	-	-	
$t = 1$	\$105,000,000	\$100,000,000 (short)	\$94,979,647 (long)	\$5,020,353
$t = 2$	\$125,000,000	\$100,000,000 (long)	\$102,280,423 (short)	\$2,283,423
$t = 3$	\$130,000,000	\$100,000,000 (short)	\$96,220,390 (long)	\$3,779,610
$t = 4$	\$150,000,000	\$100,000,000 (long)	\$100,428,337 (short)	\$428,337
Terminal	\$3,844,188,628	\$2,222,222,222 (long)	\$2,328,149,985 (short)	\$105,927,763
Value Licensing Alternative:		\$2,622,222,222 (not arbitrageable under DCF or market measure)		

CONCLUSION

- A market measure applies the RAP to the **discounted earnings price process** that confers the asset its value. Satisfying that RAP satisfies the regulatory RAP
- It is THE **LOWEST** POSSIBLE MEASURE OF AN ARM'S LENGTH RESULT AND IS THE **SAME AS A CORRECT DCF ONE** (both reflect competitive financial trading of the **asset**)
- *Calculating the PCT under a market measure does not require discount rates or a financial projection, all is needed is the price of the earnings for the year preceding valuation date; these are always available pro forma*
- *Getting that measure of the PCT allows to precisely calculate the discount rates at which to discount earnings in an application of DCF*
- TRYING TO GET THE 11.44 PERCENT AND 15.39 PERCENT DISCOUNT RATES THAT YIELD THE CORRECT RAP PRICE x_0 OF THE ASSETS USING COMPARABLES AND CAPM, SOLELY BASED ON QUALITATIVE INFORMATION ABOUT CASH FLOW RISK, IS WHAT IS TYPICALLY DONE AND LEADS TO ENDLESS CONTROVERSIES
- ELIMINATING CONTROVERSY: taxpayers can provide any subjective financial projection, reasonable or not, IT DOES NOT MATTER. A market measure does not depend on these projections, since the DCF discount rates will automatically correct for that. This is how to eliminate these endless controversies.



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Thank You





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