

Math Studies 11 – Summer review packet

Math studies assumes a certain amount of algebraic knowledge is already in place before you begin. In order to be prepared for the coming year, students should have knowledge of and work easily with the following algebra topics at a minimum:

- **Substitution**
- **Solving Linear equations**
- **Formula substitution and rearrangement**

Review the example problems and then complete the practice questions on a separate sheet of paper. Show all steps used to answer the questions. This work will be collected during the first week of school. It will be graded. If you have questions, you can review the indicated websites for help.

Algebra is a tool which we use to write mathematical ideas in a convenient way. In algebra we use letters or symbols to represent unknown quantities, or values which can vary depending on the situation.

There are several important words associated with algebra that you should be familiar with:

- $2x + 3$ is an **expression** for the quantity which is three more than twice x .
- $2x + 3 = 5$ is an **equation** which says that the quantity $2x + 3$ has the value 5. We can **solve** the equation to find the value of x .
- $2x + 3 > 5$ is an **inequality** or **inequation** which says that the value of $2x + 3$ is more than 5.
- $y = 2x + 3$ is a **formula** which connects the two **variables** x and y . If we know the value of one of the variables then we can **substitute** this value to determine the other variable. We can also **rearrange** formulae to write them in other more convenient forms.

Substitution:

<https://www.youtube.com/watch?v=d9BdbdFRZF4>

Example 1	Self Tutor	
If $p = -2$, $q = 3$ and $r = 4$, find the value of:		
a $p + 5q$	b $pr - 7q$	c $\frac{2r - 4q}{qr + p - 1}$
$\begin{aligned} a & p + 5q \\ &= (-2) + 5 \times 3 \\ &= -2 + 15 \\ &= 13 \end{aligned}$	$\begin{aligned} b & pr - 7q \\ &= (-2) \times 4 - 7 \times 3 \\ &= -8 - 21 \\ &= -29 \end{aligned}$	$\begin{aligned} c & \frac{2r - 4q}{qr + p - 1} \\ &= \frac{2 \times 4 - 4 \times 3}{3 \times 4 + (-2) - 1} \\ &= \frac{8 - 12}{12 - 2 - 1} \\ &= \frac{-4}{9} \end{aligned}$

Practice Problems:

EXERCISE 5A

1 If $l = 2$, $m = -3$ and $n = -1$, find the value of:

a $4l$

b $-n$

c $2mn$

d lmn

e $2l + m$

f $4m - 3l$

g $ml - 2n$

h $nl - 2mn$

2 If $e = 4$, $f = 2$ and $g = -3$, evaluate:

a $\frac{g}{e}$

b $\frac{e+f}{g}$

c $\frac{2g+e}{f}$

d $\frac{3f-e}{2f-g}$

e $g - \frac{e}{f}$

f $\frac{fg}{e}$

g $\frac{2g+f}{e}$

h $\frac{g-f}{e+g}$

Example 2

Self Tutor

If $x = 2$, $y = -4$ and $z = -5$, evaluate:

a y^2

b $yz^3 - 3x$

a y^2
 $= (-4)^2$
 $= 16$

b $yz^3 - 3x$
 $= (-4) \times (-5)^3 - 3 \times 2$
 $= 494$

Notice the use
of brackets.



3 If $a = 4$, $b = -1$ and $c = -3$, evaluate:

a b^2

b c^3

c $a^2 + c^2$

d $(a+c)^2$

e $a^3 + b^3$

f $(a+b)^3$

g $(2c)^2$

Example 3

Self Tutor

If $k = 5$, $l = -1$, and $m = 2$, evaluate:

a $\sqrt{k+l}$

b $\sqrt{m^2 + 3k}$

a $\sqrt{k+l}$
 $= \sqrt{5 + (-1)}$
 $= \sqrt{4}$
 $= 2$

b $\sqrt{m^2 + 3k}$
 $= \sqrt{2^2 + 3(5)}$
 $= \sqrt{19}$
 ≈ 4.36 {3 significant figures}

4 If $k = -2$, $l = 3$, and $m = 7$, evaluate:

a $\sqrt{l+k}$

b $\sqrt{m+3l}$

c $\sqrt{m-k}$

d $\sqrt{lm - 2k}$

Linear Equations:

<https://www.khanacademy.org/math/algebra-home/alg-basic-eq-ineq/alg-old-school-equations/v/algebra-linear-equations-1>

<https://www.khanacademy.org/math/algebra-home/alg-basic-eq-ineq/alg-old-school-equations/v/algebra-linear-equations-2>

Example 4

Solve for x :

a $2x - 3 = 5$

$$2x - 3 + 3 = 5 + 3 \quad \text{adding 3 to both sides}$$

$$\therefore 2x = 8$$

$$\therefore \frac{2x}{2} = \frac{8}{2} \quad \text{dividing both sides by 2}$$

$$\therefore x = 4 \quad \text{Check: } 2 \times 4 - 3 = 8 - 3 = 5 \quad \checkmark$$

b $8 - 4x = -2$

$$\therefore 8 - 4x - 8 = -2 - 8 \quad \text{subtracting 8 from both sides}$$

$$\therefore -4x = -10$$

$$\therefore \frac{-4x}{-4} = \frac{-10}{-4} \quad \text{dividing both sides by } -4$$

$$\therefore x = \frac{5}{2} \quad \text{Check: } 8 - 4 \times \left(\frac{5}{2}\right) = 8 - 10 = -2 \quad \checkmark$$

Self Tutor

The inverse of -3 is $+3$.
The inverse of $\times 2$ is $\div 2$.



Example 5

Solve for x :

a $\frac{x}{4} + 7 = 5$

$$\therefore \frac{x}{4} + 7 - 7 = 5 - 7 \quad \text{subtracting 7 from both sides}$$

$$\therefore \frac{x}{4} = -2$$

$$\therefore \frac{x}{4} \times 4 = -2 \times 4 \quad \text{multiplying both sides by 4}$$

$$\therefore x = -8 \quad \text{Check: } \frac{-8}{4} + 7 = -2 + 7 = 5 \quad \checkmark$$

b $\frac{1}{3}(x + 2) = 6$

$$\therefore \frac{1}{3}(x + 2) \times 3 = 6 \times 3 \quad \text{multiplying both sides by 3}$$

$$\therefore x + 2 = 18$$

$$\therefore x + 2 - 2 = 18 - 2 \quad \text{subtracting 2 from both sides}$$

$$\therefore x = 16 \quad \text{Check: } \frac{1}{3}(16 + 2) = \frac{1}{3} \times 18 = 6 \quad \checkmark$$

Self Tutor

EXERCISE 5B.1

1 Solve for x :

a $x + 5 = 3$	b $4x = 28$	c $-18 = -3x$	d $7 - x = 11$
e $2x + 3 = 14$	f $3x - 4 = -13$	g $5 - 2x = -9$	h $7 = 11 - 3x$

2 Solve for x :

a $\frac{x}{3} = 15$	b $\frac{1}{4}x = 16$	c $1 = \frac{x}{-3}$	d $\frac{x}{2} - 4 = 7$
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Example 6**Self Tutor**

Solve for x : $4(2x + 5) - 3(x - 2) = 16$

$$4(2x + 5) - 3(x - 2) = 16$$

$$\therefore 8x + 20 - 3x + 6 = 16 \quad \{ \text{expanding brackets} \}$$

$$\therefore 5x + 26 = 16 \quad \{ \text{collecting like terms} \}$$

$$\therefore 5x + 26 - 26 = 16 - 26 \quad \{ \text{subtracting 26 from both sides} \}$$

$$\therefore 5x = -10$$

$$\therefore x = -2 \quad \{ \text{dividing both sides by 5} \}$$

Use the **distributive law**
 $a(b + c) = ab + ac$



$$\begin{aligned} \text{Check: } & 4(2 \times (-2) + 5) - 3((-2) - 2) \\ & = 4 \times 1 - 3 \times (-4) = 4 + 12 = 16 \quad \checkmark \end{aligned}$$

Example 7**Self Tutor**

Solve for x : **a** $4x - 3 = 3x + 7$ **b** $5 - 3(-1 + x) = x$

$$\mathbf{a} \quad 4x - 3 = 3x + 7$$

$$\therefore 4x - 3 - 3x = 3x + 7 - 3x \quad \{ \text{subtracting } 3x \text{ from both sides} \}$$

$$\therefore x - 3 = 7$$

$$\therefore x - 3 + 3 = 7 + 3 \quad \{ \text{adding 3 to both sides} \}$$

$$\therefore x = 10$$

$$\text{Check: LHS} = 4 \times 10 - 3 = 37, \quad \text{RHS} = 3 \times 10 + 7 = 37. \quad \checkmark$$

$$\mathbf{b} \quad 5 - 3(-1 + x) = x$$

$$\therefore 5 + 3 - 3x = x \quad \{ \text{expanding the brackets} \}$$

$$\therefore 8 - 3x = x$$

$$\therefore 8 - 3x + 3x = x + 3x \quad \{ \text{adding } 3x \text{ to both sides} \}$$

$$\therefore 8 = 4x$$

$$\therefore \frac{8}{4} = \frac{4x}{4} \quad \{ \text{dividing both sides by 4} \}$$

$$\therefore 2 = x \text{ or } x = 2$$

$$\text{Check: LHS} = 5 - 3(-1 + 2) = 5 - 3 \times 1 = 2 = \text{RHS} \quad \checkmark$$

EXERCISE 5B.2

1 Solve for x :

- a $3(x + 2) + 2(x + 4) = 19$
- c $3(x - 3) - 4(x - 5) = 2$
- e $2(3x - 2) + 7(2x + 1) = 13$

- b $2(x - 7) - 5(x + 1) = -7$
- d $5(2x + 1) - 3(x - 1) = -6$
- f $4(x + 4) + 3(5 - 2x) = 19$

2 Solve for x :

- a $5x - 5 = 4x + 1$
- c $1 - 3x = 2x - 9$
- e $9 - 5x = x + 6$
- g $6 - x + 3(1 - x) = 7 - 2x$
- i $6 + 7x - 2(3 - x) = 5x - 8$

- b $2x - 3 = 6 - x$
- d $-4x = 8 - 2x$
- f $4x - 7 = x + 3$
- h $8 - 5(3 - x) = 9 + x$
- j $5(3x + 1) - 4x = x - 2$

3 Solve for x :

- a $5(2x - 1) + 2 = 10x - 3$

- b $2(9x - 1) = 6(3x + 1)$

c Comment on your solutions to a and b.

Formula Substitution and rearrangement:

<https://www.youtube.com/watch?v=pH7xrpsRznU>

A **formula** is an equation which connects two or more variables.

The plural of formula is **formulae** or **formulas**.

In a formula it is common for one of the variables to be on one side of the equation and the other variable(s) and constants to be on the other side.

The variable on its own is called the **subject** of the formula.

If the formula contains two or more variables and we know the value of all but one of them, we can use the formula to find the value of the unknown variable.

Step 1: Write down the formula and state the values of the known variables.

Step 2: Substitute the known values into the formula to form a one variable equation.

Step 3: Solve the equation for the unknown variable.

Example 15 **Self Tutor**

The acceleration of a falling raindrop is given by $a = g - 1.96v \text{ m s}^{-2}$ where $g = 9.8 \text{ m s}^{-2}$ is the gravitational constant and v is the speed of the raindrop.

Find:

- a** the acceleration of the raindrop before it starts falling
- b** the acceleration of the raindrop when its speed reaches 3 m s^{-1}
- c** the speed of the raindrop for which it does not accelerate.

a $a = g - 1.96v$ where $g = 9.8$ and $v = 0$

$$\therefore a = 9.8 - 1.96 \times 0$$

$$\therefore a = 9.8 \text{ m s}^{-2}$$

b $a = g - 1.96v$ where $g = 9.8$ and $v = 3$

$$\therefore a = 9.8 - 1.96 \times 3$$

$$\therefore a = 3.92 \text{ m s}^{-2}$$

c $a = g - 1.96v$ where $a = 0$ and $g = 9.8$

$$\therefore 0 = 9.8 - 1.96v$$

$$\therefore 1.96v = 9.8$$

$$\therefore v = \frac{9.8}{1.96}$$

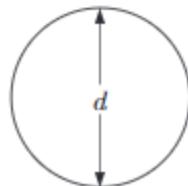
$$\therefore v = 5 \text{ m s}^{-1}$$

We use more than 3 significant figures in our working to ensure the answer is correct to at least this accuracy.

EXERCISE 5F

1 The formula for the circumference C of a circle of diameter d , is $C = \pi d$ where $\pi \approx 3.14159$ is a constant. Find:

- a** the circumference of a circle of diameter 11.4 cm
- b** the diameter of a circle with circumference 250 cm
- c** the radius of a circle of circumference 100 metres.



3 When a car travels a distance d kilometres in time t hours, the average speed for the journey is given by $s = \frac{d}{t}$ km h $^{-1}$. Find:

- the average speed of a car which travels 200 km in $2\frac{1}{2}$ hours
- the distance travelled by a car in $3\frac{1}{4}$ hours if its average speed is 80 km h $^{-1}$
- the time taken, to the nearest minute, for a car to travel 865 km at an average speed of 110 km h $^{-1}$.

4 The area of a circle of radius r is given by $A = \pi r^2$. Find:

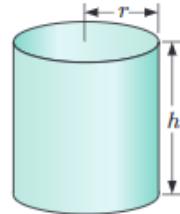
- the area of a circle of radius 5.6 cm
- the radius of a circular pond which has an area of 200 m 2 .

5 The potential difference V across an R ohm resistor is given by $V = IR$ volts, where I is the current in amps flowing through the circuit. Find:

- the potential difference across a 6 ohm resistor if the current in the circuit is 0.08 amps
- the resistance in a circuit with current 0.2 amps if the potential difference is 12 volts.

6 The volume of a cylinder of radius r and height h is given by $V = \pi r^2 h$. Find:

- the volume of a cylindrical tin can of radius 12 cm and height 17.5 cm
- the height of a cylinder of radius 4 cm if its volume is 80 cm 3
- the radius, in mm, of copper wire with volume 100 cm 3 and length 0.2 km.



G

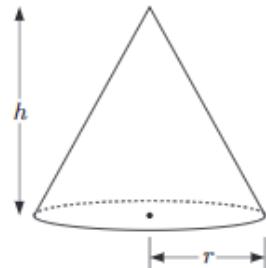
FORMULA REARRANGEMENT

Consider the formula $V = \frac{1}{3}\pi r^2 h$, which gives the volume of a cone with radius r and height h .

We say that V is the **subject** of the formula because V is expressed in terms of the other variables r and h .

The formula can be **rearranged** to make **equivalent** formulae where the other variables are the subjects:

$$h = \frac{3V}{\pi r^2} \quad r = \sqrt{\frac{3V}{\pi h}}$$



We rearrange formulae using the same methods which we used to solve equations. We perform **inverse operations** to isolate the variable we wish to make the subject.

Example 16

Self Tutor

Make y the subject of $3x - 7y = 22$.

$$\begin{aligned}
 & \text{If } 3x - 7y = 22 \\
 & \text{then } 3x - 7y - 3x = 22 - 3x \quad \{ \text{subtracting } 3x \text{ from both sides} \} \\
 & \therefore -7y = 22 - 3x \\
 & \therefore 7y = 3x - 22 \quad \{ \text{multiplying both sides by } -1 \} \\
 & \therefore \frac{7y}{7} = \frac{3x - 22}{7} \quad \{ \text{dividing both sides by } 7 \} \\
 & \therefore y = \frac{3x - 22}{7}
 \end{aligned}$$

Example 16 **Self Tutor**

Make y the subject of $3x - 7y = 22$.

$$\begin{aligned} \text{If } 3x - 7y &= 22 \\ \text{then } 3x - 7y - 3x &= 22 - 3x \quad \{ \text{subtracting } 3x \text{ from both sides} \} \\ \therefore -7y &= 22 - 3x \\ \therefore 7y &= 3x - 22 \quad \{ \text{multiplying both sides by } -1 \} \\ \therefore \frac{7y}{7} &= \frac{3x - 22}{7} \quad \{ \text{dividing both sides by } 7 \} \\ \therefore y &= \frac{3x - 22}{7} \end{aligned}$$

EXERCISE 5G.1

1 Make y the subject of:

a $x + 2y = 4$	b $2x + 6y = 7$	c $3x + 4y = 11$
d $5x + 4y = 8$	e $7x + 2y = 20$	f $11x + 15y = 38$

2 Make y the subject of:

a $x - 2y = 4$	b $2x - 6y = 7$	c $3x - 4y = -12$
d $4x - 5y = 18$	e $7x - 6y = 42$	f $12x - 13y = -44$

3 Make x the subject of:

a $a + x = b$	b $ax = b$	c $2x + a = d$
d $c + x = t$	e $7x + 3y = d$	f $ax + by = c$
g $mx - y = c$	h $c - 2x = p$	i $a - 3x = t$
j $n - kx = 5$	k $a - bx = n$	l $p = a - nx$