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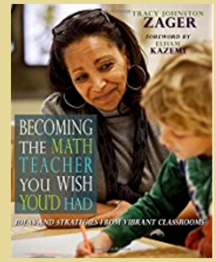


File: Mathematics Instruction

Becoming the Math Teacher You Wish You'd Had:
Ideas and Strategies from Vibrant Classrooms

By Tracy Johnston Zager (Stenhouse Publishers, 2017)

S.O.S. (A Summary of the Summary)

The main ideas of the book:

~ Real mathematicians describe math as playful, creative, and absorbing and yet students experience math in school as boring, stressful, and useless.
~ The goal of the book is to close the gap between how math is taught in schools and math as it really is by providing teachers with a deeper understanding and appreciation of the world of mathematics.

Why I chose this book:
As a former math teacher, I found this book gripping. Zager lifts the dark cloud of mathematics instruction and restores it to the fun that it should be—about wonder, exploration, and challenge. She introduces us to ten habits of real mathematicians and brings us into classrooms where math is taught in the spirit of the way mathematicians practice it.

What makes this book stand out is that Zager shows us how actual teachers implement these habits by inviting us into the most vibrant K-8 math classes with incredibly detailed and verbatim dialogue. The mathematics is deep (even in kindergarten!) and the adults and students alike are having fun! Plus, Zager includes cool math problems, strategies, websites, suggested TED talks, and resources that you will want to use immediately.

The book is cleverly organized by ten habits of real mathematicians—Mathematicians Reason, Mathematicians Connect Ideas, Mathematicians Make Mistakes, Mathematicians Ask Questions—and then grouped into five seasons so you can focus on one at a time (the book is over 350 pages!)

I recommend that school leaders stop what they're doing and buy this book for every math teacher and math leader in the building. Even if you lead a high school, all ten of the math habits are absolutely applicable to grades 9-12.

Resources
• For the tools mentioned in this book, go to: www.stenhouse.com/becomingmathteacher.
• Check out a few other recent math book summaries from The Main Idea (www.TheMainIdea - Past Book Summaries page):

Motivated (Ilana Seidel Horn)	*Making Number Talks Matter* (Cathy Humphreys and Ruth Parker)
Mathematical Mindsets (Jo Boaler)	*Five Principles of the Modern Mathematics Classroom* (Gerald Aungst)

The Scoop (In this summary you will learn…)

✓ How to bring ten habits of real mathematicians (organized into the five groups below) into math classes
✓ How mathematicians take risks, make mistakes, and are precise
✓ How mathematicians rise to a challenge, ask questions, and connect ideas
✓ How mathematicians use intuition, reason, and prove
✓ How mathematicians work together and alone

Season 1 – Breaking the Cycle & What do Mathematicians Do?

Breaking the Cycle – Chapter 1

Zager had an aha moment several years ago when her mom—who, through her business, was good at measuring, calculating, scaling up and more—said she felt she was "bad at math" and "not a math person." When Zager asked her what math was, she responded, "Math is when they hand you a sheet of paper, and it has a word problem you don't understand on it." From her mother's terrible experience in math class (something many Americans share), Zager realized that there was an enormous gap between *mathematics* and *math class*. This contrast became even more apparent as Zager spent a few years getting to know mathematicians and their field. Below are differences between how mathematicians view their work and how teachers describe their experience as math students:

Words mathematicians use to describe mathematics	*Words many teachers use to describe their experience as math students*
passion • absorbing • beauty • curiosity • play • discovery • magnificent • game • invent • persist • free • elegant • joy	memorization • stress • scary • ashamed • fear • dread • lost • difficult • useless • mistakes • fail • wrong • worksheets • humiliating • pointless

While there have been great strides in approaching math instruction in a different way—from the Common Core State Standards for Mathematical Practice to NCTM's *Practice and Standards*—we have **missed a step**. We haven't fully addressed teachers' personal histories with math instruction or their own understanding of the mathematics. We have focused more on the *program* than the *people* who are teaching it. This book aims to take a step back and invest in the people first because, "good math teaching starts with us." Zager not only introduces us to the habits of mind mathematicians actually practice, but she shows us what this can look like in real classrooms. She also provides opportunities for us to grapple with the mathematics ourselves.

What Do Mathematicians Do? – Chapter 2

From visiting dozens of math classes and paying attention to student talk, Zager can see that students often have a definitive idea of what it means to be "good" at math—you answer the teacher's questions *fast, right, easily*. In one early elementary classroom, Zager observed the teacher present a challenging problem only to hear a student say, within one to two minutes, "Oh! This is *easy!*" This is a comment most math teachers have heard. Consider the impact. Some students who are just beginning to take in the problem may feel nervous. Others just stop thinking. Still others start to believe, "Everybody else understands and I don't. I hate math."

As a result, this elementary teacher started to explore with her students what it means to be good at math. In fact, what exactly *is* math anyway? When we teach students science or history we tend to spend some time discussing what these disciplines are, but with math, we just start "doing" it. To address this, Zager and the teacher created a mini-unit, below, to explore these questions. This is something math teachers can implement in the beginning of the school year to help develop a more productive definition of math and begin to build a positive math culture. This unit helps students understand math as much more than solving computation problems.

A mini-unit that shows math is about exploring, playing, noticing, and more!

1. Use picture books to introduce students to mathematicians – Since elementary students are familiar with picture books, the teachers looked for quality picture books about the lives of mathematicians. It was hard to find books that didn't emphasize genius or brilliance. However, after searching, they found the following four books. For example, the first one shows the things Einstein loved to do, such as imagine, question, wonder, figure, read, notice, and observe.

- *On a Beam of Light: A Story of Albert Einstein* by Jennifer Berne
- *Blockhead: The Life of Fibonacci* by Joseph D'Agnese
- *The Boy Who Loved Math: The Improbable Life of Paul Erdos* by Deborah Heligman
- *Infinity and Me* by Kate Hosford

After doing some read-alouds and discussing what students observed, the teachers sent students out for a *math walk* around the school to observe, wonder, ask questions and take photos of what they saw on iPads. Students returned with photos of the gym ceiling, a stack of chairs, a census form, a timeline, a graph, and more. The teacher said, "So, you were wondering? Is that doing math?" And they recorded their thoughts on an ongoing anchor chart defining what math is about.

2. Bring math into the present day—online resources – To make sure the kids didn't develop new misconceptions (mathematicians are dead white men!) they decided to bring in current resources. You can find some of these resources at www.stenhouse.com/becomingmathteacher. They ended up showing short videos by Vi Hart, a mathemusician who has made over 100 videos doing math with fun objects like mashed potatoes and gravy. After showing the third video, *Math Improv: Fruit by the Foot*, about Mobius strips, the students got to explore with their own Mobius strips. This open-ended time to investigate patterns confused some students who were used to being shown what to do next. But it certainly challenged the idea that math was something you did by answering the teacher's questions. The class continued to watch one video a day during snack and saw that a lot of math was about problem posing, not problem solving.

3. Find math in our world – The final step of this mini-unit was for students to find math at home and bring it in for a gallery walk. This math hunt resulted in students bringing in a tube of toothpaste, a slotted pancake turner, dry macaroni, and more. In walking around the room hunting for math, students found numbers, color patterns, and geometric shapes. While those students who had traditionally been considered "strong" in math might have been a bit quieter during this unit, those who had historically held back in math came to see that there might be a place for them, too, in the math classroom. During this unit the teacher added to the definition of mathematics on the anchor chart as students' ideas changed, but it became even more important for her to continue to teach math in a way that was consistent with these ideas the rest of the year as well. She couldn't simply introduce the notion that math is about noticing, wondering, asking, and investigating for a few days and then continue with a traditional math class the rest of the year.

Season 2 – Mathematicians Take Risks, Make Mistakes, & Are Precise

Mathematicians Take Risks – Chapter 3

Too often we teach math by having our students follow steps and memorize. However, this type of "safe" approach is far from what real mathematicians do. In fact, mathematicians are most excited by math problems that are messy and have *no* answer. They walk away from the familiar and have the guts to say, "Newton died unable to solve this problem, but I think I'll give it a go." (p.31) This is the type of risk-taking we need to encourage in our classrooms. Now, you may be thinking that there's no way our students are going to tackle the mathematical concepts that Newton was unable to. But consider this example of a first-grade girl. She's walking and as she gets close to home she decides to count her steps. For fun, she counts down instead of up, so she starts with 50 and continues to 49, 48, 47, and so on. However, when she gets to zero, she's not yet home, and starts to wonder if there could be numbers on the other side of zero… She may have experienced the very same epiphany that the seventh century Indian mathematician who invented negative numbers felt years earlier. It is irrelevant to her that someone else came up with this new concept, to her it was new. In order for us to create these "aha!" moments in class, we must encourage risk—provide open problems and have students tinker, conjecture, experiment, face failure, and play.

How do we do this? Many teachers say, "I tell my students that taking risks is part of learning." However, it is not enough to simply *tell* our students, "It's OK to make mistakes in math." We need to actually *teach* them that it's OK. This chapter presents three strategies teachers can use to do this.

1. Valuing risk taking publicly – In one first-grade classroom, the teacher, Heidi, asked students to write down equations that result in 10. After giving plenty of time to work, she chose one particular student's work to project with the document camera for the class. This student's work included a number of the usual suspects (4 + 6 = 10, 7 + 3 = 10, etc.) The teacher asked students, "What do you notice?" There are a number of things Heidi did to help with risk taking. She modeled how to have a rich mathematical discussion, she gave students plenty of wait time to emphasize thought over speed, and she valued the thinking over the answers. However, a particularly valuable choice Heidi made was the decision to highlight one particular student's work. First of all, the work did not include 2 + 8 = 10, so she was modeling that an exhaustive list isn't necessarily the most valuable part of the work. Second, this student included two surprising equations, 10 = 11 – 1 and 10 = 20 – 10. Historically, students hadn't thought to use subtraction for this problem. So, instead of chiding him for forgetting 2 + 8, she publicly praised him for taking the risk of moving beyond the traditional sums and challenging himself with harder work. Furthermore, this particular student was prone to anxiety and self-confidence, so it was even more meaningful to praise him publicly for risk taking.

2. Individual written feedback – In another classroom with slightly older students, the teacher asked students to complete a similar task—write equations that result in the number 60. However, to encourage her students to take risks, she used a different strategy—*individualized written feedback* to reinforce risk taking. This particularly helps students who have always been considered "smart" because they work quickly and correctly. For example, one girl had quickly filled a whole page with simple sums to 60, 60 + 0, 59 + 1, 58 + 2, etc. The teacher wrote, "This is a lot of problems, but is it challenging to you?" The feedback was effective in encouraging the student to take risks because her next paper included equations with multiple addends and even subtraction! In the chapter, take a look at the images that show student work "before" and "after" this type of feedback.

3. Support in a vulnerable moment – In one middle school math class, the teacher creates a supportive culture and emphasizes the growth mindset from the beginning. Not only does he explicitly discuss norms and culture, but he has students, in groups, analyze and write about quotations that support a growth mindset such as: "To have a great idea, have a lot of them," "Most people don't recognize opportunity because it comes disguised as hard work" and "You can't solve math problems unless you are willing to talk to yourself." One student responded to this last quotation by writing, "I chose this [quotation] because I can understand that it means you have to help yourself understand the problem, and you have to work with yourself." This teacher further reinforces a supportive culture when students struggle or no one raises a hand by carefully choosing supportive and collaborative language such as *us, we,* and *whole room*:

"I need someone to get *us* started on it."
"Is there a volunteer to try? *We* will help."
"You've got a *whole room* of not sure, so no worries."

Mathematicians Make Mistakes – Chapter 4

Traditionally, students feel ashamed or embarrassed when they make mistakes in math class. They believe the goal is to achieve 100 percent, and those who don't must be stupid, lazy, or careless. However, this is not how mathematicians view mistakes. They see errors as golden opportunities for learning. We need to help our students develop this disposition and teach them that mistakes are not to be ashamed of, but to be learned from. Just like last chapter, it's not enough to *tell* them this, we need to teach students *how* to respond to mistakes. To handle a mistake in a productive way, we want our students to:

(1) Take the mistake in stride (2) Keep going (3) Make the most of the mistake by learning from it

This chapter highlights teachers who use strategies to teach students to respond in this way.

A supportive climate in which mistakes are normalized through language – In one fifth-grade class, most of the students had trouble with the ratio problem the teacher, Julie, presented. One student suggested a wrong answer, and then spent eight minutes listening to other students' solutions that proved hers was wrong. When Zager asked how this felt, the student responded, "Whoever gets it right and whoever gets it wrong…really, it feels kind of normal. It's not like it's out of the ordinary." When asked what the teacher does to make it feel normal, the student said she creates an environment in which everybody can share their thoughts and expects to make mistakes because they are here to learn. There are other things the teacher has done to create this environment. She chooses questions that are *open ended.* Then she allows students to engage in *productive struggle* without *rescuing them from their mistakes*. Furthermore, rather than focus on wrong and right, she starts conversations by asking for *thoughts and reasoning,* "I was curious to see how you guys would approach it" and "Does someone else want to give me their reasoning?" As a result of the culture Julie has created, students have internalized important messages about mistakes—that they are a part of learning, that it's OK for different students to come up with different approaches, and that disagreeing does not mean disrespect.

Opportunities for students to make sense of their mistakes – When students make mistakes, teachers have the option to tell them they are wrong, sweep these mistakes under the rug, and move ahead with the correct solution. However, one of the teachers highlighted in this chapter understands the value of giving students the opportunity to *spend time* with their mistakes to make sense of them. In speaking about one student who presented an incorrect solution to the class, the teacher said, "If I told her she was wrong, she would hate that! But taking her seriously, listening to her, and letting her keep working on it would make her feel important. It was more likely she would find her own mistake and own it." Because this teacher remained both supportive and neutral about the answer, not only did the student learn from her mistake, but she did so with dignity. In fact, there were three major misunderstandings in the class featured in this section, and the teacher used these three mistakes to drive her entire lesson. In addition to the curriculum and standards, teachers can use students' mistakes—particularly the rich ones—to help guide their instruction by being sure to give students plenty of opportunities to make sense of these mistakes.

Mathematicians are Precise – Chapter 5

Don't let the emphasis on play, wonder, and exploration fool you into thinking that mathematicians accept any old answers. Mathematicians deeply value thoroughness, clarity, rigor, and specificity. And yes, they do value accurate and appropriate answers. The Common Core State Standards specifically mention that it is important to "attend to precision." Precision is related to a number of concepts, including the ones just mentioned: accuracy, appropriateness, specificity, clarity, rigor, and thoroughness. This chapter presents strategies teachers can use to develop these skills and habits of mind in their students.

Strategies to raise thoroughness, clarity, rigor, and specificity – The teachers in this section have found different ways to develop these habits of mind in their students. One eighth-grade teacher, Shawn, gave students the following problem: *How many ways can you pack 24 cubes into a rectangular prism?* He led by asking, "How will you know when you have found them all?" Then he distributed worksheets with tables for them to list the length, width, and height of each prism they found. While there are only six different solutions to this problem, Shawn included *seven* rows in the chart. By providing more rows than solutions, Shawn was able to encourage his students to think through the problem in a much deeper and more thorough way. If he had only included six rows, students would have filled it out and shut down their thinking. This is a small tweak—providing more ambiguous and open-ended materials—that you can do with any problem. Consider taking a provided worksheet, and adding rows, blanks, or better yet, substitute the whole thing for a *blank* piece of paper.

Another way to promote clarity and specificity is by holding students accountable for precise language, in particular math vocabulary. The same teacher mentioned above, Shawn, has a poster with the word "it" written on it and a slash going through the word (no "it" allowed in this class). Shawn points to this poster when students make comments such as, "Looking at the graph, it looks like it went up by three every time." Rather than using "it" in this example, Shawn explains, "We're going to call variables by their names [length, width, etc.] so we know what we're talking about." Shawn also encourages precision when students talk around words. For example, when one student says, "And the angles are the same, too," Shawn asks, "There's a fancy word for that. Did you use it?" The student replies, "Congruent?" These are ways to teach math vocabulary in context which is more powerful than providing a list of words to learn or memorize.

Strategies to check for accuracy and appropriateness – As teachers, we need to teach students to actively look for and catch errors. One way to do this is through estimation. Students often learn about estimation in a single unit, and then forget to use it to check answers. In one math class, the students were told to estimate the answer to 6739 ÷ 47 without using pencil and paper. After possible answers were recorded—such as130, 155, 1524, 2600, and 6700—students chose one of these estimates and wrote why they felt it was off by using the following structure: "I think ____ is unreasonable because _____________." This is a helpful routine to get students to talk about estimation, particularly if you never end up having students calculate the actual answer. Furthermore, it helps to model what internal dialogue should sound like (*This doesn't make sense!*) so students can develop estimation as a habit to self-monitor their answers.

One teacher, Heidi, used *Buddy Checks* to help build precision and accuracy. After completing a problem, students had a peer check their work since it's easier to catch a mistake in someone *else*'s problem. However, peer review is ineffective in many classrooms. To make it successful, Heidi creates the type of class culture to ensure it is positive and actively teaches students to coach and ask questions, not give answers. For example, rather than saying, "Mabel, it's not 52, it's 82!" a student might say, "Mabel, I don't think it could be in the fifties, because I see at least 7 tens."

Season 3 – Mathematicians Rise to a Challenge, Ask Questions, & Connect Ideas

Mathematicians Rise to a Challenge – Chapter 6

In Algebra, we teach students that in a right triangle with side ***c*** as the hypotenuse, $a^2 + b^2 = c^2$. In 1967, the mathematician Pierre de Fermat wondered if this equation would work with powers greater than 2 (would $a^4 + b^4 = c^4$ hold true, for example?) He died without answering this question, and mathematicians grappled with it for over 350 years. The mathematician who eventually disproved it—spending eight years dedicated to solving this single problem—said, "Mathematicians just love a challenge. They love unsolved problems." How can we support a similar mindset in our students—that it's OK to wrestle with a problem, get stuck, and persist? Below are a few strategies to help our students become puzzlers!

Use language that frames being challenged as a *positive* thing – The language we use to frame a challenge is key. One teacher heightens excitement with words like, "I know you are all really itching for a challenge today." Once, when a student mentioned that a problem was challenging, the teacher rubbed her hands together and said, "Ooh…a challenge…cool!" The students loved this, laughed, and now any time someone mentions a *challenge* they rub their hands together, like villains, and repeat the phrase! Think about this language compared to what we often hear in math classes: "I need you to work hard in math, and then we'll do something fun," or "These problems are pretty hard. If you can't do them, that's OK. I'll help you."

Transform a textbook problem into a challenging one – Sometimes textbook problems do too much of the work for the students. For example, one textbook presented "The Mittens Problem" in which students had to figure out how many mittens four children would need in all. The issue is that the worksheet contained already drawn images of four children, a hint about the solution ("you need to find how many hands the children have all together"), a suggestion for how to solve it ("count the children by 1s and the mittens by 2s"), and a partially filled-in table like this:

Number of children	1	2		
Number of mittens	2	4		

If we want students to engage in productive struggle, but we provide scaffolding, smaller steps, tricks, rules, and algorithms, we *undermine* this goal. We particularly feel the need to "rescue" students with special needs using these types of scaffolds. When we (or the textbook) break down problems like this, *we* are doing the work, not the students. Plus, we are taking away the most important step of the problem—the need to *understand*. So, how should you approach the mittens problem? Simply give students the problem without the picture, table, and hints.

One educator who has spent a lot of time giving math problems a "makeover" is Dan Meyer. Zager insists you put down everything and watch his *Ted Talk* now: "Math Class Needs a Makeover" www.ted.com/talks/dan_meyer_math_curriculum_makeover. Meyer thinks the key is to be "less helpful." We spend too much time prescribing, demonstrating, and breaking down. We follow the *I do, you do, we do* approach to teaching. Instead, we need to rework textbook problems by first asking, what's the actual math here? Then ask, is this problem a good fit for the math I want to teach? Finally, can I open up this problem? By removing the hints, the table, and the scaffolds in the mittens problem, the teacher allows students to solve the problem in multiple ways, using various strategies, and at different levels. In fact, you could imagine using this problem for first graders who might draw or build a model all the way up to eighth graders who could use a table, draw a graph, or represent the problem algebraically. For a resource of some excellent open problems, see www.openmiddle.com to find "challenging math problems worth solving" organized by content and grade level, K-12. Plus, here's a list of criteria you might use to determine if problems are challenging yet accessible:

1. The mysterious part of the problem is mathematical.
2. The problem has very little scaffolding.
3. There are many ways to do the problem.
4. Students of different skill levels can learn from this activity.
5. The problem has natural extensions.

Mathematicians Ask Questions – Chapter 7

One mathematician said, "Computation involves going from a question to an answer. Mathematics involves going from an answer to a question." The problem is that in most math classes, students spend their time answering other people's questions rather than getting to ask their own. And there is no way to engage in good mathematics without being curious, asking questions, discovering patterns, probing, and wondering. This chapter provides several strategies for promoting curiosity in the classroom. Some strategies only require five to ten minutes, while some can be used for a whole math class or even an entire unit!

101 Questions – In the book there is a photo of an Oreo cookie next to a Double Stuf cookie. Zager asks, what's the first thing that comes to mind? Take a moment now to record a few things you might wonder looking at (or imagining) these photos. Here is some of what she wondered: *Do Double Stuf Oreos really have double the stuff? How many ways do people actually eat them? Why do people want double the stuff but not double the cookie?* The idea is to focus on questions, only on questions, not the answers. This could be a simple routine you use in class to get students into the habit of asking questions.

Notice and Wonder – This routine is similar to the strategy above, but can be helpful in preparing students to find answers to questions. You give students a math problem, but remove the question. For example:

> Ethan, Fran, and Gloria have summer jobs at the Dairy Freeze. They collect their own tips and then share them equally. One week Ethan collected $25 in tips, Fran collected $48, and Gloria collected $41.

Next, you ask, "What do you *notice*?" and "What do you *wonder*?" Without a question to distract the students, they can begin to *make sense* of the problem rather than jumping to, "Wait, is this a subtraction problem?" They slow down and start to think about the situation. They may *notice* that Ethan is getting the least amount of money. Then they might *wonder*, Why isn't he earning as many tips? Is he working in the back? Is he working fewer days? Is he grouchy? With this approach, students get out of the "get the answer quick" mode and feel less pressure because there are no right or wrong answers when you simply notice and wonder. Furthermore, when students generate enough questions by wondering, they often get to the very question the problem was going to ask.

Question-Driven Inquiry – The majority of this chapter explores what it looks like to organize an entire math unit around student questions. In one first and second-grade class, the students stumbled upon the question of whether or not "shapes are math." The teacher was going to roll out a geometry unit anyway, so she decided to follow her students' questions. There is not enough space in a summary to describe the unit here, but the teacher asked, "I would like to hear some of your questions [about shapes] so that we can figure out what we want to investigate." After recording a long list of questions, she worked with the class to narrow the list down, "Which of these questions is the most important? Which is going to help you learn the most?" What stands out is how much the teacher truly honored students' questions throughout their study of geometry. She saw her role as that of a supportive coach. As students explored questions individually and in groups, she would ask, "What do I need to get ready for you tomorrow so you can start? How can I help you? What materials are you going to need?" However, she didn't skip out to the staff room or use the questions to launch into her own unit. Nor did she sit back and let just anything happen. She continued to facilitate, guide, support and probe. Because many teachers fear what will happen if they give children this level of control, Zager includes enough detail in this chapter to paint a detailed picture of how the unit turned out.

Mathematicians Connect Ideas - Chapter 8

One famous mathematician needed to explain hyperbolic geometry to a college class. Without getting into the specifics about what this is, know that it is challenging to teach and hard for students to visualize. The mathematician—Dr. Daina Taimina—was an avid knitter and thought it might work to knit or crochet a model (there's a great image of this on p.172!) By creating this type of model, not only was she making the mathematics clearer, but she was demonstrating that thinking means making connections. Like connections literacy teachers encourage students to make (text-to-text, text-to-self, and text-to-world), the same connections can be made in math. This chapter introduces strategies to help students make math-to-self, math-to-math, and math-to-world connections.

Involve family and community members in math class - To help students connect math to the world, we can do more than give real-world problems. The problems in textbooks that attempt to do this are often phony and contrived. However, math is everywhere in life and we should look for ways to help our students make more authentic connections. One way to do this is through inviting in family members to share how they use math in their daily lives. Zager would do this at the beginning of the year and then have those parents who volunteered come in when the mathematics connected to a certain unit. For example, during the fractions unit she invited in a mother who catered out of her kitchen and they cooked together, scaling up a recipe that involved fractions to feed the entire class.

Use multiple representations and models - Too often, we teach students isolated math skills. However, many students struggle when they need to combine different math skills because they are not used to making connections. One way to help students make math-to-math connections is to have them draw visual representations of problems. For example, one teacher asked students to draw a representation of the following problem: *Darlene picked seven apples. Juan picked four times as many apples. How many apples did Juan pick?* Some students drew individual apples, others drew baskets, and others drew arrays. The teacher had the students discuss what they noticed about what was similar and different in various representations. In addition to two-dimensional representations, it also works to give students math manipulatives so they can create models. Not only do representations and models help students make sense of problems, but they also serve as tools to explain, communicate, and justify the problems to other students.

Addressing disconnections - One first-grade student incorrectly answered a few addition problems on a worksheet. For 7 + 7 she got a sum of 7. For 8 + 8 she got 8. But then the teacher asked, "Would it help if it were a story?" She gave the student a context she could relate to, "What if Emily had 7 hearts and the teacher gave her 7 more hearts?" She was able to correctly figure out this story problem. Similarly, in Brazil, a nine-year-old who works as a street vendor, was asked in school to perform 40 x 3 and she got 70. This same girl, when asked by a customer, "I'll take the three coconuts (at Cr$40 each). How much is that?" calculated correctly that it would be 120. To help our students make more math-to-self connections, we can provide them with simple story problems, have them work with objects and pictures, and regularly help them make more "human daily sense" of mathematics.

Season 4 – Mathematicians Use Intuition, Reason, & Prove

Mathematicians Use Intuition – Chapter 9

Mathematicians describe a "feel" for a problem or a "gut instinct." This does not mean they don't value accuracy or proof, but it is often a hunch or intuition that helps mathematicians begin to work with problems. Mathematics is usually a combination of a hunch about a problem and a disciplined approach to prove and test that hunch. However, in schools we tend to teach math as something that has orderly steps but leave out the intuition. While it might seem that either you are born with a *feel* for math or not, this isn't true. Students can *develop* mathematical intuition through experience and practice. As one mathematician said, "My intuitions are based on my knowledge and my experience. The more I have, the more robust my intuitions are likely to be." This chapter focuses on three key elements of developing mathematical intuition.

Building intuition around new concepts – One teacher wanted to introduce angles to her fourth-grade class, but angles can be tricky. Students get tripped up because they don't fully know if an angle is the distance between two lines or the length of the lines themselves. The teacher decided to hold off on any worksheets, and have the students play around with angles by *estimating* and *verifying* the number of degrees in different angles. The students worked with geometry manipulatives (Power Polygons) to build ninety-degree angles. By using the manipulatives, the students developed a *feel* for landmark angles such as 15, 30, 45, 60, and 90 degrees. Once students had a *sense* of what these angles looked like, they used this intuition to estimate the size of other angles.

Listening to intuition *during* problem solving – It's not enough to have students pull out their intuition at the *beginning* of a problem by making an estimate (like with the angles above) or at the *end* of a problem. Instead, we want students to deploy their intuition *throughout* the process of problem solving, otherwise they often slip back into following the steps of a procedure that may not make sense. To keep students focused on their intuition, the chapter includes over 70 questions teachers can use, such as:

Questions to redirect students to the problem while solving:

- Can you read the problem aloud again?
- Let's refresh our memories about what each of these numbers represents. What's the ____ mean?
- Wait a minute, I'm trying to visualize what's going on in this problem. Does this seem possible?

Questions to teach students to check in with their intuition during problem solving to develop a feel for mistakes:

- What tipped you off that something wasn't right?
- Does anything strike you as unreasonable here?

Regular practice with intuition – Because mathematical intuition is connected to number sense and estimation, one way to build your students' mathematical intuition is to provide regular opportunities for estimation. One middle school math teacher who blogs (Andrew Stadel, mr-stadel.blogspot) has put together a wide range of visual estimation challenges you can use with your students (www.estimation180.com) The problems provide a way to engage your students in estimating with a variety of quantities, measurements, and units. The problems are also just fun – How much do you think bacon shrinks when you cook it? How long do you think a song like "We Will Rock You" lasts? While students may start by randomly guessing at first, if you give them regular opportunities to estimate, they will start to develop some estimation skills. They will begin to use context clues and relate the situation to their prior experience. They will develop informal yardsticks (personal referents) like, a staple is about a centimeter and a box of butter weighs about a pound. One teacher helps her students develop personal referents by having a set of empty jugs in her classroom so students can visualize a pint, a quart, a gallon, a liter, etc. Then students can develop the skill of using referents to scale up an estimate. For example, if they know that people eat about a handful of green beans for dinner, they can visualize the amount of green beans needed to feed twelve people for Thanksgiving dinner.

Mathematicians Reason – Chapter 10

The previous chapter was about teaching students to develop their intuition. This and the next chapter are about the other side of the coin—learning to reason and prove. Once students have an intuition, they need to know how to subject it to reasoning and proof to determine if that instinct is valid. Unfortunately, we have not always encouraged students to develop their reasoning in math class. When students have asked questions—Why can't we divide by zero? Why is a negative multiplied by a negative a positive?—we have directed them to procedures and rules rather than developing their reasoning. This chapter introduces different aspects of reasoning we can teach students. However, note that these are not linear—just like the writing process. As one mathematician wrote, "Mathematical reasoning is an evolving process of conjecturing, generalizing, investigating *why*, and developing and evaluating arguments." Below are the four aspects of reasoning; be sure to resist the urge to turn these into a checklist!

Noticing Patterns – One of the first steps in reasoning is to *notice* patterns. Our job is to help students notice patterns (and see that this *is* math!), and then give them the opportunity to play with these patterns and test them out. For example, one student, Sheila, was playing with her calculator and found that 6 ÷ 6 = 1, 7 ÷ 7 = 1, 12 ÷ 12 = 1, and 254 ÷ 254 = 1. We can ask questions to probe so she continues to explore this pattern. There are several instructional routines that are helpful for focusing on patterns, such as **Choral Counting**. In this routine, the teacher chooses a number and has the students count forward or backward to generate a pattern:

- Starting at 8, count up by tens to 168: 8, 18, 28, etc.
- Starting at 20, count down by threes to −25: 20, 17, 14, etc.
- Starting at 0, count up by three-fourths to 12: 0, ¾, 1½, etc.

As the students count aloud together, the teacher records the responses publicly and then conducts a discussion starting with an open-ended question such as, "What do you notice?" There are over ten other questions in the book you can add to your repertoire such as, "How did you know which number would be next?" "Did someone figure out what number is next in a different way?"

Conjecturing and Generalizing – Once students start noticing patterns, they can play with them and start *conjecturing*—that is, creating a hypothesis. In the example above (with 6 ÷ 6 = 1, 7 ÷ 7 = 1, etc.), Sheila might say, "I think anything over itself is gonna be one." Then she might try this with decimals, fractions, or negative numbers to see if the conjecture holds true. Here are some questions teachers can ask to encourage students to generalize: *Do you think this will always be true? Will that work with all numbers? What would happen in the __th case?* One strategy to promote this type of thinking is **True/False Number Sentences**. Instead of having students jump into calculations, the teacher provides a few number sentences that will reveal underlying properties of operations such as the following (note that some are correct and some are wrong):

34 + 27 = 36 + 25 5 x 70 = 10 x 35 49 − 26 = 50 − 25

The discussion starts with the question, "Is it true or is it false?" and then leads to generalizations, "When you add two numbers…"

Crafting Claims – Once students are convinced their conjecture is valid, it's time to craft a *claim*. Claims are more specific than conjectures. Students need to clarify their language and specify the conditions under which their conjecture is true. For example, Sheila (from above) needs to learn how to *articulate* her claim in a way that anyone would understand, clarify the *conditions* for which the claim is true (does it work for *all* numbers?), know how to *doubt the claim,* and *revise the claim*. Here's how it might look:

> Sheila: Janae and I figured out that anything over itself is one.
> Marcus: What do you mean *anything*? What are you talking about?
> Sheila: Any number. If you divide it by itself, it's one.
> Ms. Davis: I heard you say if you divide *it* by *it*self. What are those *its*? I'm confused.
> Sheila: OK. Um, when you divide a number by itself, the answer… the, um, quotient…is one.
> Ms. Davis: Let's write Sheila's claim up here. Take some time to work together to explore this claim. Do you agree, disagree, or have a revision? (Students work in pairs and threes to try different cases and test Sheila's claim.)
> Martin: Let's try it with fractions. I wonder if we can disprove it that way.

The work that students need to do for this claim is to figure out *whether* the statement is true and if so, *when* it is true. For example, if a student comes up with the claim, "When multiplying, the product is always greater than the factors," she needs to explore the conditions under which this is true. When she tests fractions, for example, she will see that it is not always true in this case. One way to do this is through a strategy called **Always, Sometimes, Never**. In this routine, the teacher provides several mathematical statements and asks students to determine if they are true in *all* cases, *some* cases, or in *no* cases. It's helpful to draw from different areas of math in choosing statements, such as: *A square is a rectangle.* (Geometry), *300,70 = three hundred seventy* (Base Ten/The Number System), *Any number multiplied by zero equals zero.* (Operations and Algebraic Thinking).

Mathematicians Prove – Chapter 11

While there are certainly formal definitions of "proof" that mathematicians use, for use in a classroom, the most important purposes of a proof are to "convince" and "explain." Mathematicians start by trying to convince *themselves* using as much skepticism as possible. Once they have convinced themselves, they must convince *others* by explaining and justifying to help build new knowledge in others. A big part of proving something is helping yourself and others develop understanding. This definition is one that would work for schools:

Proving is convincing your skeptical peers that a mathematical statement is true in a way that helps them understand.

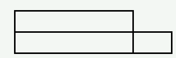
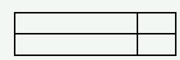
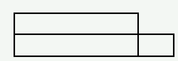
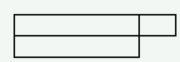
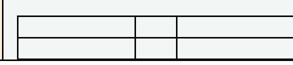
So, how do we teach proving to our students? We always need to emphasize the importance of *exploring why* when doing mathematics. One structure for exploring why is to **Convince Yourself, Convince a Friend, Convince a Skeptic**. This is a great way to think about proving something, but we need to ensure that our students are skeptical enough to do this. They need to learn that mathematical arguments are more rigorous than arguing about politics or chores. When we argue about whose turn it is to take out the trash, we are trying to convince the other person that we are right. But when we argue for a mathematical claim, we are trying to get at the truth. Therefore, we *need* another party to help us find both the strengths and the flaws in our arguments. To help students understand that criticizing a claim is *not* the same as criticizing a person, it can help to provide some sentence stems:

I respectfully disagree with _______ because ________. *I can prove ______'s claim is not true because _________.*
Another possibility is___________. *Can you clarify _____ I find it confusing because of ________.*

In class, students may try to use *inductive* or *deductive* reasoning to prove a claim. *Inductive* reasoning relies on empirical evidence to make a larger generalization: "Every time I drop something it falls to the ground. Therefore, dropped things will always fall to the ground." Note that this type of *inductive* reasoning can *never* prove a mathematical claim with total certainly. On the other hand, with *deductive* reasoning you start with a generalization and deduce a consequence: "All mammals have hair. Mice are mammals. Therefore, mice have hair." This is the type of reasoning that can be used to prove a mathematical statement with certainty.

Inductive Thinking – Although it cannot formally prove mathematical statements, *inductive* thinking helps to build reasoning and is a good place for students to start. For example, students might examine the claim, "The interior angles of a triangle sum to 180 degrees." Students might start by drawing a few triangles and measuring the angles with a protractor to find that the angles all add up to 180 degrees. Here it is helpful to ask, "Can you figure it out without a protractor or calculating? What about for any triangle—is it true?" Students may assume if several examples work then it is true. To help them see that this isn't the case, we need to find problems in which there are counterexamples to show that testing a few cases is not enough. It may be counterintuitive but we need to teach students that they should *try to be wrong*, "Is there a set of numbers for which this *won't* work? Now that we understand the pattern, let's try to break it." And then we need to use positive language when someone finds a counterexample, "Well done, Alexis, you found the hole in our argument!" Unfortunately, it can be particularly difficult to break students of their inductive thinking because people tend to rely on this type of reasoning more frequently: the sun comes up every day so the sun will always rise.

Deductive Thinking – Because this type of thinking comes less naturally, we need to ensure students get a lot of practice with it. One way to help students develop their deductive thinking is to show them three ways to make an acceptable proof in math class as displayed in the chart below: with words, algebra, or representations. The examples show three ways to demonstrate a proof of the claim "the sum of any two odd numbers is an even number."

Proof using everyday language:	Proof using algebra:	Proof using pictures:
Odd numbers are the numbers that if you group them by twos, there's one left over.	Odd numbers are the numbers of the form 2n+1, where n is a whole number.	Odd numbers are of the form: 
Even numbers are the numbers that if you group them by twos, there's none left over.	Even numbers are the numbers of the form 2n, where n is a whole number.	Even numbers are of the form: 
If you add two odd numbers, the two ones that are left over will make another group of two.	If you add two odd numbers, you get $(2k + 1) + (2m + 1) = (2k + 2m) + (1 + 1) = 2 (k + m + 1)$	Odd number plus odd number:  + 
The resulting number can be grouped by twos with none left over and, thus, is an even number.	The resulting number is of the form 2n and thus, is an even number.	= even number: 

Students may be drawn to any one of these methods—using words, representations, or symbols—as a way to conduct proofs, that is, to "convince their skeptical peers." Our job is to facilitate this process by asking probing questions, asking for different representations, modeling skepticism, and emphasizing the importance of always going back to *why*.

Season 5 –
Mathematicians Work Together and Alone & "Favorable Conditions" for All Math Students

Mathematicians Work Together and Alone – Chapter 12

In an unlikely pairing of mathematicians, an American woman and a Russian man, during the Cold War, together were able to solve a great unsolved problem known as Hilbert's Tenth. While mathematicians are not *told* to work in pairs or get into groups with assigned roles, they *choose* to spend some of their time working alone and some of their time seeking interactions with other mathematicians. If we want our students to know when, how, and why to interact with each other mathematically, we need to *teach* them how to do this. This does not mean simply putting students into collaborative groups. We need to teach students to specify what kind of help they want from others and how to say when they need independent work time first, "I need to work on this problem on my own for a while before I can talk about it." This chapter describes four of the most productive types of mathematical interactions: Thinking Partnerships, Cross-Pollination, Math Disputes, and Peer Feedback.

Thinking Partners – This type of interaction occurs when students have opportunities to collaborate with peers, sitting side-by-side, in a non-competitive and mutually respectful climate. The stakes are low and students serve as sounding boards as they try to solve a rich mathematical problem. This last part is key—if the problem is a straight-forward one that students can solve on their own, then they don't need assistance from their peers. There is no reason complicate things with group dynamics if students can solve the problem on their own. So, if we want students to practice interacting mathematically, we need to provide the types of problems for which students will seek out other students. A few research studies have furthered our thinking when it comes to student collaboration. One study showed that *where* students work and the *surfaces* they work on make an enormous difference. In the study, students were much more productive when given erasable whiteboards than permanent surfaces like paper. With the whiteboards, they were more willing to jump into the problem, participate, discuss, and persist. In addition, this same study showed that students were more engaged and eager when standing and writing on vertical surfaces than when sitting and writing on horizontal ones. Finally, this researcher found that students were more effective in groups when those groups were randomly assigned by the teacher, but only when the teacher drew the popsicle sticks *in front of* students to make the groups. When teachers created these groups ahead of time or when students chose their own groups, the groups were much less productive.

Cross-Pollination – Cross-pollination is when students listen to and build on the ideas of others. To do this, students have time to work on a problem alone for a while and then have the opportunity to see or hear what others have done to help them enhance their own thinking. One common strategy to achieve this is the **Gallery Walk**. Students work on a problem and then create a poster or whiteboard to represent their work. After a while, students put up their work and students walk around and look at each poster. Teachers provide guidelines such as, "As you walk around, find someone who used a similar strategy to yours and then someone who used a different strategy and jot down both in your notebook." Afterwards, it is essential that students are given time to revise their own work with the ideas they have gathered from others.

Math Disputes – Argument lies at the heart of mathematics. It is through arguments, reasoning, and logic that we arrive at the truth. We need to teach our students not to take it personally when we argue; it's actually a way of partnering together to solve a problem. One strategy to teach students the benefit of argument is called **Take a Position** or **Vote with your Feet**. After giving students a problem, you ask them to take a stand and vote with their feet. For example, one teacher gave students a problem that involved a game with spinners and asked who thought the game was fair for the two players and who thought one player had an advantage. Students moved to one side of the room if they thought it was fair and another if they thought it was unfair, and then clarified their arguments. One teacher, Chris Luzniak, teaches students how to conduct debates in class. He takes closed math problems like 18 x 32 and turns them into more debatable questions like, "What is the easiest way to solve 18 x 32 using mental math?" Then he teaches his students to present their arguments by creating a *claim* (a controversial statement) and a *warrant* to back it up (reason why your controversial statement is true). This way, his students learn not only to create arguments, but to critique mathematical reasoning as well.

Peer Feedback – This final type of interaction, peer feedback, allows students to get someone with a different perspective to look at their work, help clarify or tighten their work, or check for correctness. One way we can structure peer feedback is for students to give it to individual students. One teacher has her students provide feedback on cards and gives them the opportunity to write a question, what impressed them, something they disagree with and why, something they hadn't thought of before, or any advice. Another way to promote peer feedback is to do a group critique by projecting one student's work for the rest of the class to see and comment on. This chapter captures the productive dialogue that occurred in one group critique that lasted 23 minutes!

"Favorable Conditions" for All Math Students – Chapter 13

Clarence F. Stephens is a math educator that few know about, but who made tremendous improvements in the way math was taught when he served as the mathematics department chair at State University of New York (SUNY) Potsdam from 1969 to 1987. He was able to open up math to far more than the high-achieving students. How? By creating the most favorable conditions as was written in the mission statement of his department:

> "The study of pure mathematics can be undertaken successfully by a large number of students if they are provided with a supportive environment including: careful and considerate teaching by a well-trained and dedicated faculty, continual encouragement, successful (student) role models, enough success to develop self-esteem, enough time to develop intellectually, recognition of their achievement, and the belief that the study is a worthwhile endeavor. We are dedicated to providing this supportive environment." (p.352)

Our students come to us in kindergarten full of wonder, curiosity, observations, and mathematical ideas. Then they learn to dislike math in school. We can prevent this by creating the most *favorable conditions*--as outlined in this book--for all students so they can enjoy mathematics now and for the rest of their lives.

I could create so many PD activities from the inspiring ideas in this book! Below is just a sampling of ideas.

I. What is Math?

In order for students to see math as much more than quick and correct calculations, teachers need to have a broader idea of what math is. After they broaden their own views of math, they can prepare a mini-unit to introduce this expanded vision to their class.

1. **Gallery Walk:** Start off with a gallery walk. Put each of the following quotations on a different poster and hang them throughout the room. Leave out Post-it notes and markers for teachers to write and post their reactions to these quotations.

> *It is impossible to be a mathematician without being a poet in the soul.* –Sofia Kovalevskaia
>
> *Pure mathematics is the world's best game. It is more absorbing than chess, more of a gamble than poker, and lasts longer than Monopoly. It's free. It can be played anywhere—Archimedes did it in a bathtub.* --Richard J. Trudeau
>
> *The true spirit of delight…is to be found in mathematics as surely as poetry.* --Bertrand Russell
>
> *The life of a mathematician is dominated by an insatiable curiosity, a desire bordering on passion to solve the problems he is studying.* --Jean Dieudonné
>
> *All mathematicians share…a sense of amazement over the infinite depth and the mysterious beauty and usefulness of mathematics.* --Martin Gardner
>
> *The way of mathematics is to make stuff up and see what happens.* --Vi Hart

2. **Discussion:** Next, gather as a large group and discuss the following:
- What were some of the teachers' reactions to the above quotations?
- How did the teachers experience math when they were in school?
- What is the difference between *mathematics* and *math class*?
- Did their idea of what math is change at all?
- What does it mean to be "good" at math?

Be sure to let teachers know that Tracy Zager designed her book around important habits of *real* mathematicians that we can bring into the classroom. Share the chapter headings with the teachers:

Mathematicians: take risks, make mistakes, are precise, rise to a challenge, ask questions, connect ideas, use intuition, reason, prove, and work alone and with others.

3. **Create a mini-unit, *What is Math?*** After teachers have stretched their own ideas about what math is, they can begin to think of a unit to introduce these broader ideas to their students. Share the structure of the sample mini-unit in Chapter 2 so they can follow and adapt it. Give teachers time to gather resources like the ones below. Tell teachers that for this unit they will co-construct an anchor chart that defines math with their class over a few days, adding to the definition after introducing each of the activities below:

1. Use Picture Books to Introduce Students to Mathematicians – The idea is to find books that do *not* paint mathematicians as "geniuses," but rather show their love of wonder and curiosity. Provide copies of the following and let teachers poke around the Internet to find books they can introduce to their own students:
- *Blockhead: The Life of Fibonacci* by Joseph D'Agnese
- *Infinity and Me* by Kate Hosford
- *On a Beam of Light: A Story of Albert Einstein* by Jennifer Berne
- *The Boy Who Loved Math: The Improbable Life of Paul Erdos* by Deborah Heligman

2. Bring Math into the Present Day—Online Resources – Give teachers time to watch short videos by Vi Hart, a mathemusician who has made over 100 videos doing math with fun objects like mashed potatoes and gravy. Let them choose a few of these videos that they can share with students to open up their ideas about math.

3. Find Math in Our World – Tell teachers that the next task is for students to find math at home and bring it to class or find it around the school and take photos of it in order to have a gallery walk. Have teachers jot down questions they will use to conduct the discussion after this gallery walk.

4. Co-create Norms for Math Class – Now that everyone has some fresh ideas about math, it's time to create new *math* norms that reflect this new understanding. Share the sample norms at Youcubed.org: https://bhi61nm2cr3mkdgk1dtaov18-wpengine.netdna-ssl.com/wp-content/uploads/2017/03/Norms-Poster-2015.pdf and have teachers plan how they'll develop math norms of their own.

II. Take Risks, Make Mistakes, and Be Precise

1. **Video:** Have teachers watch a 6-minute video that is about a teacher *using* student mistakes to help the class learn - "My Favorite No" –- https://www.teachingchannel.org/videos/class-warm-up-routine. Next, discuss what the teacher had to do *up until* this point to be able to conduct this activity successfully and what she does *during* the activity to make it successful.

2. **Brainstorm and Choose Words:** In order for students to take risks and make mistakes, we need to create a culture of safety. Other than telling students, "Take a risk" or "Failure is OK," brainstorm additional ways to support risk taking in the math classroom.

One of the ways to do this is through our words – we can use language to normalize mistakes and encourage risk. Have teachers look at some of the typical language teachers use in math and have them re-write how they might rephrase these lines.

Current Language	Language that promotes safety in risk taking and mistake making: ask about reasoning, not answers; ask more open-ended questions; value productive struggle and effort; emphasize the learning that comes from mistakes
"Who knows the answer?" "What is 8x12?" "That's not the answer. Does anyone else have it?"	

3. **Practice Individualized Feedback to Encourage Risk Taking**: Give teachers a piece of paper and have them spend a few minutes writing down number sentences with a solution of 10 (like 1 + 9 = 10). Next, have everyone pass their paper to the right. Now, ask teachers to try to write a comment on the paper that does *not* focus on which sums to 10 are missing or which equations are wrong, but rather, that will encourage the person to take more risks and think of *other* types of equations (such as $\sqrt{100} = 10$, 90 – 80 = 10, etc.) Give the work back to the original owner and now have them add to their list of equations. Discuss: What was the effect of the risk-encouraging comments as opposed to what would have happened if the person simply corrected your errors?

III. Rise to a Challenge, Ask Questions, and Connect Ideas

1. **Video**: Have teachers watch Dan Meyer's 11-minute TED Talk "Math Class Needs a Makeover" to help teachers think about ways to open up and improve textbook math problems: https://www.ted.com/talks/dan_meyer_math_curriculum_makeover. Discuss.

2. **Create or Find Richer Math Problems**: Next, have teachers practice by taking a few of their own textbook problems and tweaking them to make them more challenging. Alternatively, they can poke around www.openmiddle.com which is organized by grade level and math topic, and find some new problems that would fit with their curriculum. Finally, have them determine if the problems they have tweaked/chosen fit the criteria of a rich problem:

1. The mysterious part of the problem is mathematical.
2. The problem has very little scaffolding.
3. There are many ways to do the problem.
4. Students of different skill levels can learn from this activity
5. The problem has natural extensions.

3. **Notice and Wonder:** Tell teachers this is an excellent strategy to get students to think through a problem before rushing to an answer. Take a problem and erase the question, like this sample from the book, and ask students what they *notice* and *wonder*:

> Ethan, Fran, and Gloria have summer jobs at the Dairy Freeze. They collect their own tips and then share them equally. One week Ethan collected $25 in tips, Fran collected $48, and Gloria collected $41.

Give the teachers a few minutes to practice this strategy themselves. Have them jot down what they *notice* and *wonder*. For example, they might write, "I *notice* that the girls earn more than the boy. I *wonder* if girls are more industrious?" Or "I *notice* that Ethan didn't earn as much. I *wonder* if Ethan worked fewer hours?" Next have teachers share their thoughts with the larger group. Ask the group to discuss how they think this might be beneficial in having students solve problems.

IV. Use Intuition, Reason, and Prove

1. **Would You Rather:** To help students develop their intuition, they need lots of practice looking at the world, using estimation, and testing their answers. Have teachers go through three steps to do this so that they can then lead their students through it later.

1) Give teachers a "Would you rather" question from http://www.wouldyourathermath.com/ (For example, would you rather have 364 jellybeans and give 188 to friends, or have 281 jellybeans and give 137 to friends?) Have them state which they'd rather do.
2) Next, have teachers use mental math (no pencils!) to estimate the answer and explain their reasoning.
3) Finally, have teachers use pencil and paper to prove if they were right or not.

2. **True/False? Make a Claim.** Again, put teachers in the position of students so they can conduct a similar activity with students. Give teachers a few number sentences and ask them to state which are true. For example:

a) 8 x 4 = 16 x 2 b) 8 x 10 = 16 x 20 c) 12 x ½ = 24 x 1

Next, have them come up with a claim or a generalization. For example, when you multiply two numbers, if you… then you… Conduct an in-depth discussion with the teachers about their claim. Is this *always* true? For decimals? For negative numbers? Can you use representations (pictures) to show this? Can you use symbols (algebra) to show this? How do we know?